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5

Content

内容

光学

Review

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Chapter 1

Title 几何光学(定理) **Date** /

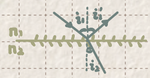
Keywords 关键词 | Notes 笔记 | Review 复习记录 / / / / /

6. 基本定理

▸【Theorem】 1. 光的直线传播　光在均匀介质中沿直线传播

2. 反射定律　反射角 = 入射角

3. 折射定律　$n_1 \sin i_1 = n_2 \sin i_2$



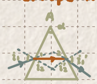
【Theorem】 $n_1 < n_2$ 时 n_1 对应光疏介质　n_2 对应光密介质　$i_1 > i_2$

* 从光密进入光疏时　$\sin i_1 \geq \frac{n_2}{n_1}$　折射消失　发生全反射　临界角 $i_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

▸【Def】 由于不同波长的光在介质中的传播速度不同，折射角随波长而异的现象，称为色散

通常棱镜折射率 n 随入↑而↓，紫光偏折最大，红光偏折最小

【Prop】 最小偏向角求折射率



几何关系　$\delta = \angle 1 + \angle 2 = (i_1 - i_2) + (i_1' - i_2')$

$= (i_1 + i_1') - (i_2 + i_2') = i_1 + i_1' - \alpha$

折射关系　$n = \frac{\sin i_1}{\sin i_2} = \frac{\sin i_1'}{\sin i_2'}$　$\Longmapsto n = \frac{\sin\frac{\delta_{min}+\alpha}{2}}{\sin\frac{\alpha}{2}}$

为使偏向角 δ 最小　$\frac{d\delta}{di_1} = 1 + \frac{di_1'}{di_1} = 0$　$\longmapsto \frac{1-\sin^2 i_1}{n^2 - \sin^2 i_1} = \frac{1-\sin^2 i_1'}{n^2 - \sin^2 i_2}$

得 $i_1 = i_1'$　$i_2 = i_2'$　$\longmapsto$ 测出 δ_{min}　i_1 则有 $i_1 = \frac{1}{2}(\delta_{min} + \alpha)$

▸【Theorem】 Fermat 原理　光从A传播到B时，沿传播所需时间取极值的路线传播　$\delta t = \delta\int_A^B \frac{ds}{v} = 0$

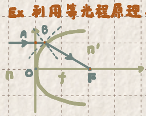
$t_{A\to B} = \int_A^B dt = \frac{1}{c}\int_A^B n\,ds$

【Remark】 n与S的乘积为光程

【Remark】 Fermat 原理可表述为使光程取极值

▸【Theorem】 Fermat 原理　光路可逆

Ex 利用等光程原理求介质曲线方程



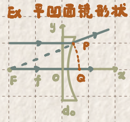
选取 ABF 路径　光程 $n \times \overline{AB} + n' \times \overline{BF}$

选取 OF 路径　光程 $n' \times \overline{OF}$

记 $B(x, y)$　$nx + n'\sqrt{(f-x)^2 + y^2} = n'f$

解得 $(n'^2 - n^2)x^2 + n'^2 y^2 - 2n'(n-n')fx = 0$

Ex 平凹面镜形状



光线1　$n \times \overline{CP} + \overline{PF}$

光线2　$n \times \overline{OA} + \overline{AB} + \overline{BF}$

$\longmapsto nx = nd + \sqrt{(x^2+f)^2 + y^2} - (f+d)$

$\Longmapsto (1-n^2)x^2 + y^2 - 2(n-1)(f+nd) = (n-1)^2 d^2 - 2(n-1)df$

Summary 总结

Chapter I

Title 几何光学（成像）　　Date /

Keywords 关键词	Notes 笔记	Review 复习记录 / / / / /

§. 单球面成像

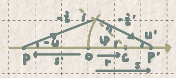
折射

▸ [Prop] 笛卡尔坐标规则

(i) 光线从左侧进入光学系统

(ii) 坐标原点　右正左负　上正下负　顺正逆负　(字母为坐标 可+可-)

▸ [Theorem] Abbe 不变式 $\frac{n'}{s'} - \frac{n}{s} = \frac{n'-n}{r}$



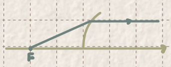
(1) $n\sin(-i) = n'\sin(-i')$

(2) ΔPMC: $-i = (-u) + \varphi$　　$\Delta P'MC$: $-i' = \varphi - u'$

(3) $-u \approx \frac{h}{-s}$　　$u' \approx \frac{h}{s'}$　　$\varphi \approx \frac{h}{r}$

▸ [Def] 定义 光焦度 $D = \frac{n'-n}{r}$

▸ [Theorem] 焦点坐标公式　物方 $f = \frac{n}{n-n'} r$　　像方 $f' = \frac{n'}{n'-n} r$

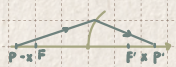


$s' = \infty$　　$-\frac{s}{n} = \frac{r}{n'-n}$

$s = -\infty$　　$\frac{s'}{n'} = \frac{r}{n'-n}$

[Remark] $\frac{f}{f'} = -\frac{n}{n'}$

▸ [Theorem] Gauss 成像公式 $\frac{f'}{s'} + \frac{f}{s} = 1$

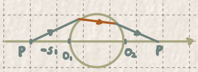


$-s = -f - x$

$s' = f' + x'$

▸ [Theorem] Newton 成像公式　$xx' = ff'$

Ex　$n = 1.5$　R　(i) 平行光成像？　(ii) 物在球前 2R 处



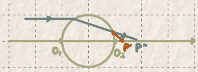
(i) $\frac{n'}{s'} - \frac{n}{s} = \frac{n'-n}{r}$

$s_1 = -\infty$　$r_1 = +R$　$n_1 = 1$　$n_2 = 1.5$

$\longmapsto$ $s_1' = 3R$

$s_1 = +R$　$r_2 = -R$　$n_1 = 1.5$　$n_2 = 1$

$\longmapsto$ $s_2' = \frac{R}{2}$



(ii) $\frac{n'}{s'} - \frac{n}{s} = \frac{n'-n}{r}$

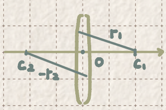
$s_1 = -2R$　$r_1 = +R$　$n_1 = 1$　$n_1' = 1.5$

$\longmapsto$ $s' = +\infty$

$s_2 = \infty$　$r_2 = -R$　$n_2 = 1.5$　$n_2' = 1$

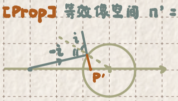
$\longmapsto$ $s_2' = 2R$

[Remark] 薄透镜焦距公式 $\frac{1}{f'} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$



反射

▸ [Prop] 等效像空间 $n' = -n$



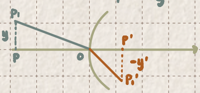
$\frac{n'}{s'} - \frac{n}{s} = \frac{n'-n}{r}$

[Theorem] 反射成像公式 $\frac{1}{s} + \frac{1}{s'} = \frac{2}{r}$

Keywords 关键词 | Notes 笔记

§. 放大率

▸ [Def] 线放大率 $\beta = \frac{y'}{y}$



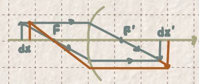
$n\sin i = n'\sin i'$

$\sin i \approx \frac{y}{-s}$　　$\sin i' \approx \frac{-y'}{s'}$

$\mapsto \beta = \frac{y'}{y} = \frac{ns'}{sn'}$

$\beta > 0$ 正立虚像　$\beta < 0$ 倒立实像

▸ [Def] 轴向放大率 $\alpha = \frac{dx'}{dx} \geq 0$

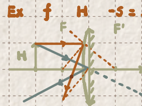


$xx' = ff'$

$\mapsto \alpha = -\frac{x'}{x}$

$\alpha = \frac{n'}{n}\beta^2$

Ex　f　H　$-s = 2f$　求像的位置和大小



$\frac{f'}{s'} + \frac{f}{s} = 1$

$(f' = -f > 0)$

1° 第一次　$s = -2f'$　$s' = 2f'$

$\beta_1 = \frac{s'}{s} = -1$

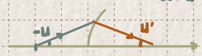
2° 第二次　$s = 2f'$　$s' = -2f'$

$\beta_2 = \frac{ns'}{n's} = 1$　→ $\beta = -\frac{1}{3}$

3° 第三次　$s = 2f$　$s' = \frac{2}{3}f'$

$\beta_3 = \frac{s'}{s} = \frac{1}{3}$

▸ [Def] 角放大率 $\gamma = \frac{\tan u'}{\tan u}$

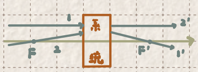


$\gamma = \frac{s}{s'} = \frac{x}{f'} = \frac{f}{x'}$

▸ [Theorem] $\alpha\gamma = \beta$；多系统放大率可累乘

§. 共轴球面系统

基点法



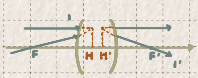
(i) 焦点：与平行光共轭的点

(ii) 主点：$\beta = 1$ 的一对共轭平面与光轴的交点　物方主点与像方主点

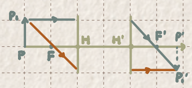
(iii) 节点：$\gamma = 1$ 的一对共轭平面与光轴的交点

焦点 → 主点 → 节点

(1) 寻找主点



(2) 分别以两个主点作为物、像空间的原点



(1) 寻找节点

$\gamma = \frac{x}{f'} = \frac{f}{x'} = 1$

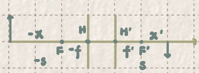
$x_N = f'$　$x_{N'}' = f$ (相对于 F F')

$s_N = s_{N'}' = f + f'$ (相对于 H H')

(2) 同介质时 $f = -f'$

此时 $s_N = s_{N'}' = 0$

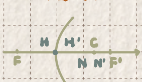
▸ [Theorem] Gauss 公式 Newton 公式在基点法中仍适用



$$\begin{cases} \frac{f}{s} + \frac{f'}{s'} = 1 \\ xx' = ff' \end{cases}$$

基点的确定

(i) 单球面折射



(1) $f = \frac{nr}{n-n'}$　$f' = \frac{n'r}{n'-n}$

(2) $\beta = -\frac{f}{x_H} = -\frac{x_{H'}'}{f'} = 1$

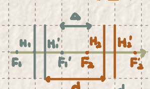
$x_H = -f$　$x_{H'}' = -f'$

(3) $\gamma = \frac{x}{f'} = \frac{f}{x'} = 1$

$x_N = f'$　$x_{N'}' = f$

对单球面 H H' 与 O 重合　N N' 与 C 重合

(ii) 两个子系统组合



引入光学间隔 Δ　主点间隔 d

$d = \Delta + f_1' - f_2$

(1) $L_{HH_1} = f_1\frac{d}{\Delta}$　$L_{H'H_2'} = f_2'\frac{d}{\Delta}$

(2) $f' = -\frac{f_1'f_2'}{\Delta}$　$f = \frac{f_1f_2}{\Delta}$

(3) $n = n'$　节点与主点重合

Summary 总结

▸ [Summary] 单透镜基础公式

Abbe 不变式 $\frac{n'}{s'} - \frac{n}{s} = \frac{n'-n}{r}$　焦点坐标公式 $f = \frac{nr}{n-n'}$　$f' = \frac{nr}{n'-n}$

Gauss / Newton 公式 $\frac{f}{s} + \frac{f'}{s'} = 1$ / $xx' = ff'$

普适公式　(1) $\frac{f}{f'} = -\frac{n}{n'}$　(2) $\frac{f}{s} + \frac{f'}{s'} = 1$　(3) $xx' = ff'$

放大率公式　$\beta = \frac{ns'}{n's} = -\frac{f}{x} = -\frac{x'}{f'}$　$\gamma = \frac{s}{s'} = \frac{x}{f'} = \frac{f}{x'}$

基点公式　(1) $L_{HH_1} = f_1\frac{d}{\Delta}$　(2) $f = \frac{f_1f_2}{\Delta}$　(3) $s_N = s_{N'}' = f + f'$

$L_{H'H_2'} = f_2'\frac{d}{\Delta}$　$f' = -\frac{f_1'f_2'}{\Delta}$

Chapter 1

Title 几何光学（扩充页） Date /

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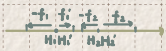
6.基点法示例

Ex d r_1 r_2 n 求 f f'

$$\begin{cases} f_1 = \frac{nr_1}{1-n} \\ f_1' = \frac{r_1}{n-1} \end{cases} \quad \begin{cases} f_2 = \frac{r_2}{n-1} \\ f_2' = \frac{nr_2}{1-n} \end{cases}$$



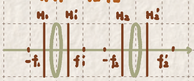
$d = d \qquad \Delta = d - f_1' + f_2$

1° $L_{HH_1} = f_1 \frac{d}{\Delta} \qquad L_{H'H_2'} = f_2' \frac{d}{\Delta}$

2° $f = \frac{f_1 f_2}{\Delta} \qquad f' = -\frac{f_1' f_2'}{\Delta}$

$\overset{n}{\longmapsto} \begin{cases} \frac{1}{f'} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2} + \frac{(n-1)d}{nr_1r_2}\right) \\ \end{cases}$

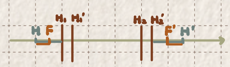
Ex $f_1 f_1'$ $f_2 f_2'$ d 求组合焦距 f f'



对其进行基点分析 $\Delta = d - f_1' + f_2$

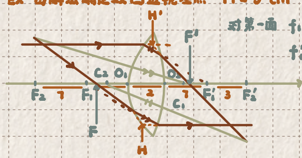
(1) $L_{H_1H} = f_1 \frac{\Delta}{d} = f_1 \frac{d - f_1' + f_2}{d}$

$L_{H_2'H'} = f_2' \frac{\Delta}{d} = f_2' \frac{d - f_1' + f_2}{d}$



(2) $f = \frac{\Delta}{f_1 f_2} = \frac{d - f_1' + f_2}{f_1 f_2} \qquad f' = -\frac{d}{f_1' f_2'}$

Ex 图解法确定双凸透镜基点 $r_1 = 3\text{ cm}$ $r_2 = -5\text{ cm}$ $d = 2\text{cm}$ $n = 1.5$



对第一面 $f_1 = \frac{r_1}{1-n} = -6\text{cm}$ $\qquad f_1' = \frac{nr_1}{n-1} = 9\text{cm}$

对第二面 $f_2 = \frac{nr_2}{n-1} = -15\text{cm}$ $\qquad f_2' = \frac{r_2}{1-n} = 10\text{cm}$

Summary 总结

Chapter 1

Title 几何光学（仪器） Date /

Keywords 关键词	Notes 笔记 Review 复习记录 / / / / /

6. 常用光学仪器

眼睛

▸ [Def] 近点：水晶体被压缩最厉害时能看清的最近点

远点：睫状肌放松时能看清的最远点

▸ [Def] 视度 = 1 / 远点距离 L (m) 近视度 = 视度 × 100

放大镜

▸ [Def] 放大镜放大率 $M = \frac{\tan\theta'}{\tan\theta} = \frac{25}{f'(cm)}$

显微镜

▸ [Prop] 由物镜和目镜组成（物镜 f_o 短 目镜 f_o 长）

物镜放大像以便目镜观察 $\beta_o = -\frac{x'}{f_1'} \approx -\frac{\Delta}{f_1'}$ 对目镜 $M_e = \frac{25}{f_2'}$

则总放大率 $M = -\frac{\Delta}{f_1'}\frac{25}{f_2'}$

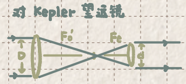
望远镜

▸ [Prop] 物镜 f_o 较长 口径大 目镜 f_e 较短 口径小

物镜 F_1' 与 目镜 F_2 重合

存在两种望远镜 Kepler（两个凸透镜） Galilei（一凸一凹）

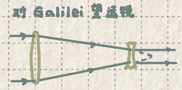
对 Kepler 望远镜



角放大率 $M = \frac{\tan u'}{\tan u} = \frac{f_o'}{f_e} = -\frac{f_o'}{f_e'}$ 倒立

几何关系 $\frac{f_o}{f_e} = \frac{D}{d}$

对 Galilei 望远镜



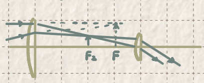
角放大率：$M = \frac{\tan u'}{\tan u} = \frac{f_o'}{f_e}$ 正立

几何关系：$\frac{f_o}{f_e} = \frac{D}{d}$

$M = \frac{D}{d}$

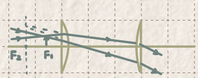
存在两种目镜 Huygens 目镜 Ramsden 目镜

对 Huygens 目镜



$f_1 : f_2 : d = 3:1:2$

对 Ramsden 目镜



$f_1 : f_2 : d = 3:3:2$

Summary 总结

Keywords 关键词 | Notes 笔记 | Review 复习记录 / / / / /

§. 基本公式

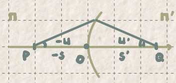
▸ [Prop] (i) 折射公式 $n_1 \sin i_1 = n_2 \sin i_2$ $n_0 \equiv \frac{c}{v_0}$ 密 → 疏 $i_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

// 色散 $n = n(\lambda)$ 红光偏折 < 紫光 // 最小偏向角：$i_1 = \frac{1}{2}(\delta_{min} + \alpha)$ $n = \dfrac{\sin\frac{\alpha+\delta_{min}}{2}}{\sin\frac{\alpha}{2}}$

(ii) Fermat 原理 $t_{A\to B} = \frac{1}{c}\int_A^B n\,ds$ // 等光程原理求透镜方程

§. 单球面问题

▸ [Def] 笛卡尔坐轴



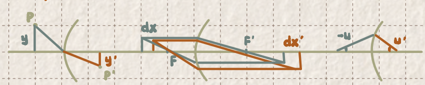
注意：u, u', s, s' → 坐标数字, 可正可负

图中标注为物理实值, 全正未式

代入公式时应代入坐标

[Remark] 一类特殊折射：反射 定义 $n' \equiv -n$

放大率



线：$\beta = \frac{y'}{y}$ 轴*：$\alpha = \frac{dx'}{dx}$ 角：$\gamma = \frac{\tan u'}{\tan u}$

▸ [Theorem] 几何光学公式库

A. 涉及曲率：

(i) Abbe 不变式 $\frac{n'}{s'} - \frac{n}{s} = \frac{n'-n}{r}$

(ii) 焦点坐标公式 $\begin{cases} f = \frac{n}{n-n'}r \\ f' = \frac{n'}{n'-n}r \end{cases}$

B. 不涉及曲率：

(i) 焦距关系：$\frac{f}{f'} = -\frac{n}{n'}$

(ii) Gauss / Newton 公式：$\begin{cases} \frac{f}{s} + \frac{f'}{s'} = 1 \\ xx' = ff' \end{cases}$

C. 放大率衡量：

(i) $\beta \equiv \frac{y'}{y} = \frac{s'}{n'}\cdot\frac{n}{s}$

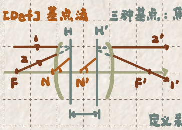
(ii) $\alpha \equiv \frac{dx'}{dx} = -\frac{x'}{x}$

(iii) $\gamma \equiv \frac{\tan u'}{\tan u} = \frac{u'}{u} = \frac{s}{s'}$

(iv) $\beta = \alpha\gamma$

§. 基点法

▸ [Def] 基点法 三种基点：焦点 F、主点 H、节点 N



F：平行光汇聚

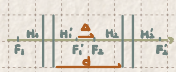
H：$\beta = 1$

N：$\gamma = 1$ $n = n'$ 时 N 与 H 重合

定义新坐标原点 H、H'

▸ [Theorem] 单球面 H、H' 与 O 重合 N N' 与 C 重合

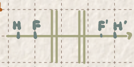
▸ [Prop] 系统合并



→ ：正 ← ：负

Δ：光学间隔

d：主点间隔



D. 基点法成像公式：(新坐标系下)

(i) Gauss 公式：$\frac{f}{s} + \frac{f'}{s'} = 1$

(ii) Newton 公式：$xx' = ff'$

E. 系统组合公式

(o) $d = \Delta + f_1' - f_2$

(i) $L_{HH_1} = f_1\frac{d}{\Delta}$ $L_{H'H_2'} = f_2'\frac{d}{\Delta}$

(ii) $f = \frac{f_1 f_2}{\Delta}$ $f' = -\frac{f_1' f_2'}{\Delta}$

(iii) $S_N = S_{N'} = f + f'$

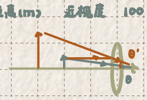
§. 光学仪器

眼睛

定义：近点 远点 视度 = 1 / 远点距离(m) 近视度 100 × 视度 明视距离：25 cm

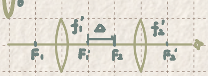
放大镜

放大率 $M = \frac{\tan\theta'}{\tan\theta} = \frac{25}{f'(\mathrm{cm})}$



显微镜

短 f 物镜 + 长 f 目镜 $M = -\frac{\Delta}{f_1'}\frac{25}{f_2'}$



望远镜

长 f 大 D 物镜 + 短 f 小 D 目镜 F_1' 与 F_2 重合 $M = \frac{D}{d}$

Summary 总结

Title 光的干涉 Date /

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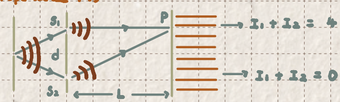
§. 干涉现象

▸[Prop] 双缝干涉



(1) 场的线性叠加 $\vec{E}(P,t)=\sum_{n=1}^{N}\vec{E}_n(P,t)$

(2) 波强度的叠加 $\vec{E}_m(P,t)=\vec{E}_{m_0}(P)\cos[wt+\varphi(P)]$

$\vec{E}(P,t)=\vec{E}_1(P,t)+\vec{E}_2(P,t)$ $I(P)=\langle\vec{E}\cdot\vec{E}^*\rangle=\langle(\vec{E}_1+\vec{E}_2)\cdot(\vec{E}_1^*+\vec{E}_2^*)\rangle$

$\Longleftrightarrow I=I_1+I_2+\boxed{2\vec{E}_{10}\cdot\vec{E}_{20}\langle\cos\delta\rangle}$ ← 干涉项 δ: 位相差

$\delta=(w_1-w_2)t+\varphi_1(P)-\varphi_2(P)$

▸[Prop] 为使干涉项 $2\vec{E}_{10}\cdot\vec{E}_{20}\langle\cos\delta\rangle\neq0$ 需满足:

(1) 振动方向不垂直 $\vec{E}_{10}\not\perp\vec{E}_{20}$

(2) $w_1=w_2$

(3) 初始位相差 $[\varphi_1(P)-\varphi_2(P)]$ 恒定

同一光源可满足

▸[Prop] $\vec{E}_1(P,t)=\vec{E}_{10}(P)e^{i[wt-\frac{2\pi}{\lambda}r_1]}$

$\vec{E}_2(P,t)=\vec{E}_{20}(P)e^{i[wt-\frac{2\pi}{\lambda}r_2]}$

$I(P)=I_1+I_2+2\vec{E}_{10}\cdot\vec{E}_{20}\cos[\frac{2\pi}{\lambda}(r_2-r_1)]$

$=I_1+I_2+2\vec{E}_{10}\cdot\vec{E}_{20}\cos[\frac{2\pi}{\lambda_0}\boxed{n(r_2-r_1)}]$ → 光程差 Δ

$=I_1+I_2+2E_{10}\cdot E_{20}\cos\delta$ $\delta=\frac{2\pi}{\lambda}\Delta$ as $L\gg d$

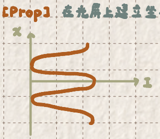
$=I_1+I_2+2\sqrt{I_1I_2}\cos\delta$

▸[Prop] 亮纹 $\delta=2k\pi$ 即 $\Delta=k\lambda$ $I_{max}=I_1+I_2+2\sqrt{I_1I_2}$

暗纹 $\delta=(2k+1)\pi$ 即 $\Delta=(k+\frac{1}{2})\lambda$ $I_{min}=I_1+I_2-2\sqrt{I_1I_2}$

$I_1=I_2=a^2$ 时 $I=4a^2\cos^2\frac{\delta}{2}$ $I_{max}=4a^2$ $I_{min}=0$

▸[Prop] 在光屏上建立坐标轴 $I=I_1+I_2+2\sqrt{I_1I_2}\cos[\frac{2\pi}{\lambda}n(r_2-r_1)]$



$r_2-r_1=\sqrt{L^2+(x+\frac{d}{2})^2}-\sqrt{L^2+(x-\frac{d}{2})^2}\approx\frac{x}{L}d$

$\Delta=nd\frac{x}{L}$ $\Longrightarrow$ $x_{k,亮}=\frac{L}{nd}k\lambda$

$x_{k,暗}=\frac{L}{nd}(k+\frac{1}{2})\lambda$

条纹间距 $e=x_{k+1}-x_k=\frac{L}{nd}\lambda$

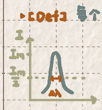
▸[Def] 定义对比度 $V\equiv\frac{I_{max}-I_{min}}{I_{max}+I_{min}}$

对双缝干涉 $V=\frac{2\sqrt{I_1I_2}}{I_1+I_2}\xlongequal{I_1=I_2}1$

§. 光的相干性

时间相干性

▸[Def] 每个原子不同时刻发出的光波之间的相干性



$\Delta\lambda$ 称为波包的线宽 ($\Delta\nu$ 对应频宽) $\Delta\lambda$ $\Delta\nu$ 越小, 单色性越好

能产生干涉的最大光程差值为相干长度 L_c

要求 $L_c>\Delta$ $L_c=c\tau$ 相干时间 $\tau=\frac{L_c}{c}=\frac{1}{\Delta\nu}$

▸[Prop] 由于 $x_{k,亮}=\frac{L}{d}k\lambda$ λ_1 短波 λ_2 长波 $\Delta\lambda=\lambda_2-\lambda_1$

错开间距 $\Delta x=\frac{L}{d}k\Delta\lambda$

[Remark] 当 $\Delta x=e$ 时 $V=0$ $\frac{L}{d}k\Delta\lambda=\frac{L}{d}\lambda$ 即 $k=\frac{\lambda}{\Delta\lambda}$

$L_c=\frac{\lambda^2}{\Delta\lambda}=\frac{c}{\Delta\nu}$

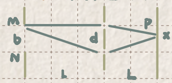
Keywords 关键词

Notes 笔记

§. 实现干涉的途径

空间相干性

▸[Def] 不同位置发出的光波之间的相干性

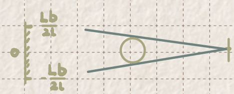


$\Delta_{M,P} = \frac{dx}{L} + \frac{db}{2L} = k\lambda$

$\Longrightarrow x_{亮k} = \frac{L}{d}k\lambda - \frac{Lb}{2L}$ $\quad x_{M0} = -\frac{Lb}{2L}$

$\Delta_{N,P} = \frac{dx}{L} - \frac{db}{2L} = k\lambda$

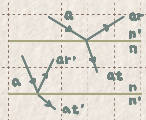
$\Longrightarrow x_{亮k} = \frac{L}{d}k\lambda + \frac{Lb}{2L}$ $\quad x_{N0} = \frac{Lb}{2L}$



错开距离 $\xi = \frac{Lb}{L}$ $\quad \xi / e = 1$ 时 $V = 0$ $\quad b_c = \frac{L}{d}\lambda$ $\quad d_c = \frac{L}{b}\lambda = \frac{\lambda}{\varphi}$

▸[Prop] 将一个光源的一束光波分割成多束, 并使其重新相互交会 并在交会区域产生干涉条纹 波前分割法 振幅分割法

薄膜干涉



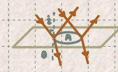
定义反射系数 $r \equiv E_{反} / E_{入}$

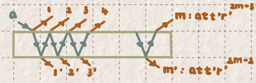
透射系数 $t \equiv E_{透} / E_{入}$

$$\begin{cases} r = -r' \\ tt' + r^2 = 1 \end{cases}$$

反射率 $R \equiv \frac{I_{反} A\cos i}{I_{入} A\cos i} = r^2$

透射率 $T = 1 - R$





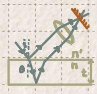
$I_{(1)} = |ar|^2$

$I_{(2)} = |att'r'|^2 \quad \cdots$

$I_{(3)} \ll I_{(2)}$ 只考虑前两束

$V = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} = \frac{2T}{1+T^2} = \frac{2R}{1+(1-R)^2}$

$V' = \frac{2\sqrt{I_1' I_2'}}{I_1' + I_2'} = \frac{2R}{1+R^2} \approx 2R$



光程差 $\Delta_r = n\frac{2t}{\cos\theta} - 2n' t \tan\theta \sin i - \frac{\lambda}{2}$

$= \frac{2nt}{\cos\theta} - 2nt\tan\theta\sin\theta - \frac{\lambda}{2}$

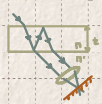
$= 2nt\cos\theta \boxed{-\frac{\lambda}{2}} \longrightarrow$ 半波损失

// 亮条纹 $\Delta_r = k\lambda$ 即 $2nt\cos\theta = (k+\frac{1}{2})\lambda$

暗条纹 $\Delta_r = (k+\frac{1}{2})\lambda$

即 $2nt\cos\theta = (k+1)\lambda$

[Remark] 光疏进入光密的反射时 存在半波损失



光程差 $\Delta_t = 2nt\cos\theta$

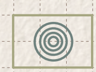
// 亮 $\Delta_t = k\lambda$ 即 $2nt\cos\theta = k\lambda$

暗 $\Delta_t = (k+\frac{1}{2})\lambda$ 即 $2nt\cos\theta = (k+\frac{1}{2})\lambda$

1. 等倾干涉

$\Delta_r = 2nt\cos\theta - \frac{\lambda}{2}$ while t const

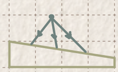
同一级干涉条纹由相同入射角的光束干涉形成



2. 等厚干涉

$\Delta_r = 2nt\cos\theta - \frac{\lambda}{2}$ while θ const

同一级干涉条纹由相同厚度处的光束干涉形成



Summary 总结

Title 光的干涉

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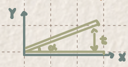
§. 其它干涉模型

楔形膜干涉

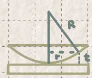


$\Delta r = 2nt\cos\theta - \frac{\lambda}{2}$ $t = \alpha x$ $n = 1$

$\Rightarrow \Delta r = 2\alpha x\cos\theta - \frac{\lambda}{2}$

定义条纹周期 $\frac{dx}{dk} = \frac{\lambda}{2\alpha\cos\theta}$ 反映条纹疏密

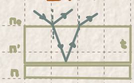
牛顿环干涉



$\Delta r = 2nt\cos\theta - \frac{\lambda}{2}$ $t = R - \sqrt{R^2 - r^2} \overset{R \gg r}{=\!=} \frac{r^2}{2R}$ $n = 1$ $\cos\theta = 1$

$\Rightarrow \Delta r = \frac{r^2}{R} - \frac{\lambda}{2}$

增透膜与高反膜

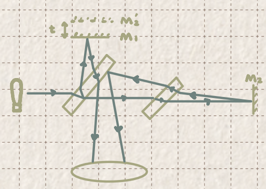


$\Delta r = 2n't\cos\theta$

正入射时 $\cos\theta = 1$

增透 $\Delta r = (k + \frac{1}{2})\lambda$ 高反 $\Delta r = k\lambda$

迈克尔逊干涉仪



$\Delta r = 2nt\cos\theta$ $\delta = \frac{2\pi}{\lambda}\Delta r$ 空气 $n = 1$

可控制 $I_1 = I_2$ 时 $V = 1$ $I = 4a^2\cos^2\frac{\delta}{2}$

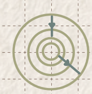
$M_1 \parallel M_2'$ 时 等倾干涉

$M_1 \nparallel M_2'$ 时 等厚干涉

对等倾干涉 $\Delta = 2t\cos\theta_k = k\lambda$

1) $t\downarrow$ $\theta_k\downarrow$ 条纹内收

$t\uparrow$ $\theta_k\uparrow$ 条纹外冒 $(r \propto \theta)$

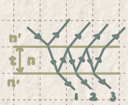


2) t 不变 $\frac{dk}{d\theta_k} = -\frac{2t\sin\theta_k}{\lambda}$

$\theta\uparrow$ $\lambda\downarrow$ $t\uparrow$ 条纹变密

§. 多光束干涉

▸ [Prop] 振幅分割的多光束干涉

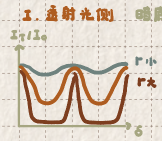


对等倾干涉，相邻光束光程差 $\Delta = 2nt\cos\theta$，$\delta = \frac{2\pi}{\lambda}\Delta = \frac{4\pi nd\cos\theta}{\lambda}$

复振幅 1: att' 2: $att'r^2e^{-i\delta}$ 3: $att'r^4e^{-i2\delta}$ ...

总振幅 $y = \sum_{m=0} att'r^{2m}e^{-im\delta} = \frac{att'}{1 - r^2e^{-i\delta}}$

Ⅰ. 透射光侧 暗底亮纹

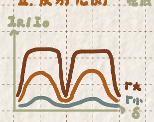


$I_T = y \cdot y^* = \frac{a^2(tt')^2}{1 + r^4 - 2r^2(1 - 2\sin^2\frac{\delta}{2})} = \frac{I_0}{1 + F\sin^2\frac{\delta}{2}}$ 锐度系数 $F = (\frac{2r}{1-r^2})^2$

$\delta = 2k\pi$ $I_T = I_{max} = I_0$

$\delta = (2k+1)\pi$ $I_T = I_{min} = \frac{I_0}{1+F}$ r 越大 F 越大 V 越大

Ⅱ. 反射光侧 亮底暗纹



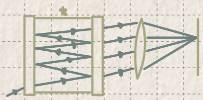
$I_0 = I_T + I_R$

得 $I_R = I_0\frac{F\sin^2\frac{\delta}{2}}{1 + F\sin^2\frac{\delta}{2}}$

Keywords 关键词

Notes 笔记

Fabry - Perot 干涉仪

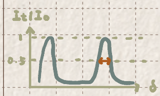


$I_t = I_0 \frac{1}{1+F\sin^2\frac{\delta}{2}}$ $\delta = \frac{2\pi}{\lambda} 2t\cos\theta$

亮 $I = I_{max}$ $\delta = 2k\pi = \frac{2\pi}{\lambda} 2t\cos\theta$

k const $\lambda\uparrow$ $\theta\downarrow$ 越靠内

λ const $k\uparrow$ $\theta\downarrow$ 越靠内



条纹半宽度 $\frac{I_t}{I_0} = \frac{1}{1+F\sin^2\frac{\delta}{2}} = \frac{1}{2}$ 即 $F\sin^2\frac{\delta}{2} = 1$

$\delta_{1/2} = \pm 2\arcsin\frac{1}{\sqrt{F}} + 2k\pi$ $\Delta\delta = 4\arcsin\frac{1}{\sqrt{F}}$

$\Delta\delta \approx \frac{4}{\sqrt{F}}$ as $F \gg 1$

▸[Def] 定义条纹精细度 $\mathcal{F} \equiv \frac{2\pi}{\Delta\delta} = \frac{\pi\sqrt{F}}{2}$ $\sim 10 - 10^2$

(1) 自由光谱程 考虑一段连续谱 $\lambda_1 \sim \lambda_2$ $\Delta\lambda = \lambda_2 - \lambda_1$

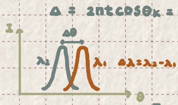
$\Delta = 2nt\cos\theta_k = k\lambda$ ($\lambda\uparrow$ $k\downarrow$)

当 $k\lambda_2 = (k+1)\lambda_1$ 时 干涉条纹消失 $\Delta\lambda = \frac{\lambda_1}{k}$ 即 $\Delta\lambda$ 需满足 $\Delta\lambda \le \frac{\lambda_1}{k}$

为使从中心起 干涉条纹便出现 自由光谱程 $\Delta\lambda_{FSR} = \frac{\lambda_1\lambda_2}{2t} \approx \frac{\lambda^2}{2t}$ $\Delta\lambda \ll \Delta\lambda_{FSR}$

(2) 色分辨能力

$\Delta = 2nt\cos\theta_k = k\lambda$ $\longleftrightarrow$ $\Delta\theta = -\frac{k}{2nt\sin\theta}\Delta\lambda$



与此同时 相位半宽度 $\Delta\delta = \frac{4}{\sqrt{F}} = -\frac{4\pi}{\lambda} nt\sin\theta\,\Delta\theta$

角半宽度 $\Delta\theta = -\frac{\lambda}{\pi nt\sin\theta}\frac{1}{\sqrt{F}}$

临界时 λ_1、λ_2 条纹不可分辨 $\Delta\lambda_{min} = \frac{2\lambda}{k\pi\sqrt{F}}$

色分辨本领 $\frac{\bar{\lambda}}{\Delta\lambda_{min}} = k\frac{\pi\sqrt{F}}{2} = k\mathcal{F}$ 中心区域分辨本领越强

Summary 总结

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Title 光的干涉章末总结

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§. 干涉原理

▸【Theorem】光强 $I \propto$ 振幅 E^2 $\quad I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$

其中 位相差 $\delta = \frac{2\pi}{\lambda}\Delta$ $\quad \Delta$: 光程差

【Prop】干涉极大 $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$ $\quad$ 干涉极小 $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$

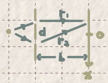
对比度 $V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$

下列所示不同模型即是通过不同手段构造出 δ，即 Δ

§. 双光束干涉模型

// comment: 统一分析亮条纹

杨氏双缝干涉（波前分割）

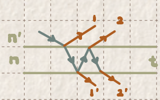


$\Delta = (r_2 - r_1)n = \frac{nd}{L}x$

亮条纹 $\Delta = \frac{nd}{L}x = k\lambda$ 即 $x = \frac{L\lambda}{nd}k$

条纹间距 $e = \frac{dx}{dk} = \frac{L\lambda}{nd}$

薄膜干涉（振幅分割）



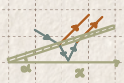
反射率 $R = r^2$ $\quad$ 折射率 $T = 1 - R$

光疏 → 光密的反射 ∃ 半波损失

1-2 干涉 $\Delta_r = 2nt\cos\theta - \frac{\lambda}{2} = k\lambda$

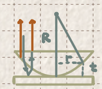
1'-2' 干涉 $\Delta_t = 2nt\cos\theta = k\lambda$

楔形膜干涉 与 牛顿环干涉（等厚干涉）



$\Delta_r = 2\alpha x\cos\theta - \frac{\lambda}{2} = k\lambda$

条纹周期 $\frac{dx}{dk} = \frac{\lambda}{2\alpha\cos\theta}$



$\Delta_r = \frac{r^2}{R} - \frac{\lambda}{2} = k\lambda$

非均匀等距

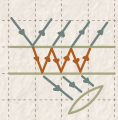
迈克尔逊干涉仪 *（不考）

根据 M_2 摆放状态，可产生等倾、等厚干涉 $\quad \Delta_r = 2t\cos\theta = k\lambda$

§. 多光束干涉

// comment: 统一考虑透射面

Fabry - Perot 干涉仪



$I_R = \frac{I_0 F\sin^2\frac{\delta}{2}}{1 + F\sin^2\frac{\delta}{2}}$ $\quad$ 亮底暗纹

$I_T = \frac{I_0}{1 + F\sin^2\frac{\delta}{2}}$ $\quad$ 暗底亮纹 $\quad \delta = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda}2t\cos\theta$

一些系数：锐度系数 $F = \left(\frac{2r}{1-r^2}\right)^2$ $\quad$ 条纹半宽度 $\Delta\delta = 4\arcsin\frac{1}{\sqrt{F}}$ $\quad$ 条纹精细度 $\mathcal{F} = \frac{\pi\sqrt{F}}{2}$

自由光谱程 $\Delta\lambda_{FSR} = \frac{\lambda^2}{2t}$ $\quad$ 色分辨本领 $\frac{\bar{\lambda}}{\Delta\lambda_{min}} = k\mathcal{F}$

Summary 总结

Title 光的衍射

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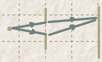
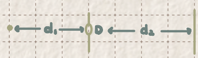
§. 衍射现象

▸ [Def] 当光源及观察屏到衍射物距离有限远时　发生 Fresnel 衍射　　Else

当光源及观察屏到衍射物距离无限远时　发生 Fraunhofer 衍射　　$d_1, d_2 \gg \frac{D^2}{\lambda}$



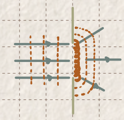


▸ [Theorem] Huygens 子波原理

(i) 光波的波前上每一点可被看成一个新的次级波源，发出子波

(ii) 下一时刻的波前为所有子波共同的包络面

(iii) 波的传播方向为从子波源指向子波面和包络面的切点连线方向



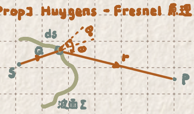
▸ [Theorem] Fresnel 子波叠加原理

(i) 所有子波数学上线性叠加

(ii) 各子波贡献大小由其权重因子决定

§. Fresnel 衍射

▸ [Prop] Huygens - Fresnel 原理数学表示



在波面 Σ 上取小面元 dS　对 P 点光场贡献为 $du_Q(P,t)$

$du_Q(P,t) \propto \frac{e^{i(\omega t-kr)}}{r}$　/ $\propto u_0(Q)$　/ $\propto K(\theta_0,\theta)$　/ $\propto dS$

$\mapsto du_Q(P,t) \propto u_0(Q)K(\theta_0,\theta)\frac{e^{i(\omega t-kr)}}{r}$

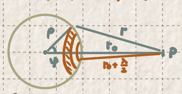
$u(P,t) = E(P)e^{i\omega t}$　　$E(P) = A\oiint_\Sigma u_0(Q)\frac{e^{-ikr}}{r}K(\theta_0,\theta)\,dS$

$\begin{cases} A = \frac{i}{\lambda} \\ K(\theta_0,\theta) = \frac{\cos\theta_0+\cos\theta}{2} \end{cases}$　　考虑球面 Σ_0，$\theta_0=0$　$K(\theta) = \frac{1+\cos\theta}{2}$

$K(0)=1$　　$K(\pi)=0$　　排出无后退波

Fresnel - Kirchhofer 衍射积分　　$E(P) = \frac{i}{\lambda}\oiint_\Sigma u_0(Q)\frac{1+\cos\theta}{2}\frac{e^{-ikr}}{r}\,ds$

[Remark] 半周期带　考虑各向同性空间中的一个单色点光源 S_0



$u_0(Q) = \frac{a}{\rho}e^{ik\rho}$　　$K(\theta) = \frac{1+\cos\theta}{2}$

此时 $E(P) = \sum_{n=1}^{m} E_n$　　$E_n(P) = \frac{i}{\lambda}\int_{r+(n-1)\frac{\lambda}{2}}^{r+n\frac{\lambda}{2}} u_0(Q)K(\theta)\frac{e^{-ikr}}{r}\,dS$

令 $dS = 2\pi(\rho\sin\psi)\rho\,d\psi$　代入 $E_n(P)$

$\mapsto E_n(P) = (-1)^{n+1}a\frac{2K(\theta_n)}{\rho+r_0}e^{-ik(\rho+r_0)}$

$\begin{cases} K(\theta_n) \text{ 随 } \theta_n\uparrow \text{ 而 } \downarrow \quad |E_{n+1}| < |E_n| \\ \text{相邻半周期带对 P 点场强贡献相反} \end{cases}$

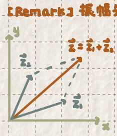
$|E_P| = |E_1| - |E_2| + |E_3| - \cdots \pm |E_m|$

m 取奇时 $|E_P| = \frac{|E_1|}{2} + \frac{|E_m|}{2}$

m 取偶时 $|E_P| = \frac{|E_1|}{2} - \frac{|E_m|}{2}$

一般情况下 $|E_m| = 0$　　$|E_P| = \frac{|E_1|}{2}$

[Remark] 振幅矢量图

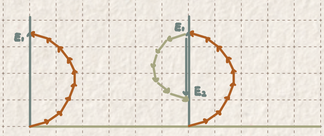


$z = \rho e^{i\psi} = \rho\cos\psi + i\rho\sin\psi$

对单个半周期带的振幅矢量图

N 个子带，位相差 π/N

$\vec{E}_1 = \sum_{m=1}^{N} e^{-ik(r_0+\frac{m}{N}\frac{\lambda}{2})}$



依此类推　$|E_P| = \frac{|E_1|}{2} \pm \frac{|E_m|}{2} \approx \frac{|E_1|}{2}$

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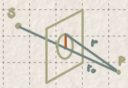
圆孔 Fresnel 衍射



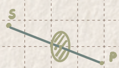
圆孔外半周期带被遮挡 $E_p = \sum_{n=1}^{L} E_n = \frac{|E_1|}{2} \pm \frac{|E_L|}{2}$

P 在轴上移动 $\longmapsto$ 半周期带数目变动

$L =$ 奇 变亮 $L =$ 偶 变暗 $L = 1$ 最亮 $L = 2$ 最暗

$L = \infty$ $E_p = \frac{E_1}{2}$

圆屏 Fresnel 衍射



圆屏半周期带被挡 $E_p = \sum_{n=L}^{m} E_n = \frac{|E_L|}{2} \pm \frac{|E_m|}{2}$

$m - L =$ 奇 取负 $m - L =$ 偶 取正

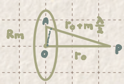
$m \to \infty$ $E_p = \frac{|E_m|}{2}$ 即 $I_p = \frac{I_n}{4}$ 泊松亮点

Fresnel 波带板

挡住偶数带 $|E_p| = |E_1| + |E_3| + \cdots + |E_{2n+1}|$

挡住奇数带 $|E_p| = |E_2| + |E_4| + \cdots + |E_{2n}|$

振幅型带 $|E_p| = |E_1| + |E_2| + \cdots + |E_n|$



$R_m^2 = (r_0 + m\frac{\lambda}{2})^2 - r_0^2 = m r_0 \lambda + \frac{m^2\lambda^2}{4} \cong m r_0 \lambda$ as $m \ll \frac{4r_0}{\lambda}$

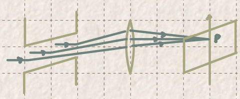
通常情况下 $r_0 \sim 10\ cm$ $\lambda \sim 550\ nm$ $m \ll 10^6$

$R_m = \sqrt{m r_0 \lambda} \propto \sqrt{m}$ 定义 $f = r_0 = \frac{R_m^2}{m\lambda} \propto \frac{1}{\lambda}$ 比透镜色差相反

$m\frac{\lambda}{2} = r - r_0 = \sqrt{r_0^2 + R^2} - r_0 \cong \frac{1}{2}\frac{R^2}{r_0} \propto \frac{1}{r_0}$ 定义次焦距 $\frac{f}{3}$ $\frac{f}{5}$ …

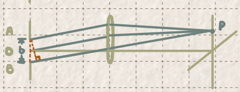
§. Fraunhofer 衍射

▸ [Prop] 单缝衍射



$E(P) = A \iint_{\Sigma} u_0 \frac{e^{-ikr}}{r} K(\theta)\, ds$ $\theta \to 0$

$= A \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{e^{-ikr}}{r} dy = A \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{e^{-i\frac{2\pi}{\lambda}(r_0 - y\sin\theta)}}{r_0 - y\sin\theta} dy$



$= E_0 \frac{\sin(\frac{\pi}{\lambda} b\sin\theta)}{\frac{\pi}{\lambda} b\sin\theta} = E_0 \frac{\sin\beta}{\beta}$ as $\beta = \frac{\pi b_0 \sin\theta}{\lambda}$

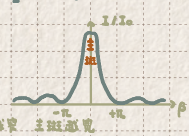
▸ [Remark] β 物理含义 $\beta = \frac{\delta_{AB}}{2}$ AB 到 P 点相位差的一半

$I_p = I_0 \frac{\sin^2\beta}{\beta^2}$ $\frac{dI}{d\beta} = \beta\sin\beta(\beta\cos\beta - \sin\beta) = 0$

[i] $\beta = 0$ 中央主极大 $I_p = I_0$

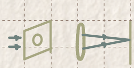
[ii] $\sin\beta = 0$ 且 $\beta \neq 0$ 极小 $I_p = 0$ $\beta = k\pi$

[iii] $\beta = \tan\beta$ 次极大 $I_p = \cos^2\beta\, I_0$



主极大区域宽度 $y'_{\pm1} = f\tan\theta_{\pm1} \cong \pm\frac{f\lambda}{b}$ $d = \frac{2f\lambda}{b}$ 缝越窄 主斑越宽

▸ [Prop] 圆孔衍射



$I_p = I_0 \left(\frac{2J_1(\beta)}{\beta}\right)^2$ $\begin{cases} \beta = \frac{\pi D\sin\theta}{\lambda} \\ J_1:\text{第一类1阶 Bessel 函数} \end{cases}$

[i] $\beta = 0$ $\sin\theta = 0$ $I = I_0$ 主极大

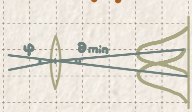
[ii] $\beta = \pm 3.83$ $\sin\theta = \pm\frac{1.22\lambda}{D}$ $I = 0$ 极小

Airy 斑 $\theta_{min} = \arcsin\left(\frac{1.22\lambda}{D}\right) \cong 1.22\frac{\lambda}{D}$ 第一级极小暗条纹

$L_{min} = f\tan\theta_{min} \cong 1.22\frac{f\lambda}{D}$

§. 成像系统分辨本领

▸ [Theorem] Reyleigh 判据 两个点像能辨别开的最小距离是 Airy 斑的半径



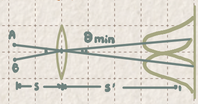
$\Delta\varphi_{min} = \theta_{min} \cong \frac{1.22\lambda}{D}$

$\Delta L_{min} = \theta_{min} f$

分辨本领 $\equiv \frac{1}{\Delta\varphi_{min}} / \frac{1}{\Delta L_{min}} \propto D$

Keywords 关键词 | Notes 笔记

显微镜的分辨本领



最小像点间距 $y'_{min} \cong \theta_{min} s' = \frac{1.22\lambda}{D} s'$

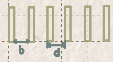
$yn\sin u = y'n'\sin u'$ // $n'=1$

得 $y_{min} = \frac{0.61\lambda}{n\sin u} = \frac{0.61\lambda}{NA}$ → 数值孔径，通常<2

[Remark] 增大显微镜分辨能力途径　增大 n；增大 u；减小 λ

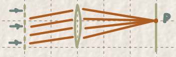
§. 衍射光栅

▸ [Def] 大量相同的狭缝等平行排列而构成的周期性衍射元件称光栅



d：光栅周期

b：缝宽



P点光场为所有狭缝的贡献之和　对第m个单缝 $E_P^m = E_0 \frac{\sin\beta}{\beta} e^{i\phi_m}$　$\beta = \frac{\pi b\sin\theta}{\lambda}$

$E_P = \sum_{m=1}^{N} E_P^m = E_0 \frac{\sin\beta}{\beta}(1 + e^{-i\delta} + e^{-2i\delta} + \cdots + e^{-(N-1)i\delta})$　$\delta = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda} d\sin\theta$

$= E_0 \frac{\sin\beta}{\beta} \frac{\sin N\gamma}{\sin\gamma}$　$\gamma = \frac{\delta}{2} = \frac{\pi d\sin\theta}{\lambda}$

▸ [Theorem] $I_P = I_0 \frac{\sin^2\beta}{\beta^2} \frac{\sin^2 N\gamma}{\sin^2\gamma}$　单缝衍射因子：$\frac{\sin^2\beta}{\beta^2}$　多光束干涉因子：$\frac{\sin^2 N\gamma}{\sin^2\gamma}$

[Prop] 对干涉因子 $\frac{\sin^2 N\gamma}{\sin^2\gamma}$　$\frac{d\cdot}{d\gamma} = \sin\gamma\sin N\gamma(N\sin\gamma\cos N\gamma - \cos\gamma\sin N\gamma) = 0$

[1] $\gamma = k\pi$　$\frac{\sin^2 N\gamma}{\sin^2\gamma} = N^2$　主极大 ⟼ 光栅方程 $d\sin\theta = k\lambda$

[2] $\gamma = \frac{m\pi}{N}$　$\frac{\sin^2 N\gamma}{\sin^2\gamma} = 0$　主极小 ⟼ $d\sin\theta = \frac{m\lambda}{N}$　$\frac{m}{N} \notin \mathbb{Z}$

[3] $N\tan\gamma = \tan N\gamma$　次极大

两个主极大间有 $N-1$ 个主极小　$N\uparrow$　$I\uparrow$　尖锐↑

[Prop] 对衍射因子 $\frac{\sin^2\beta}{\beta^2}$　干涉极大与衍射极小重合：缺级现象

$$\begin{cases} d\sin\theta = k\lambda \\ b\sin\theta = k'\lambda \end{cases} \longmapsto \frac{d}{b} = \frac{k}{k'}$$

衍射中央主极大条纹数 $2(\frac{d}{b})-1$

光栅参数

$d\sin\theta = k\lambda$

[1] 色散率　$\frac{d\theta}{d\lambda} = \frac{1}{d}\frac{k}{\cos\theta}$

[2] 色分辨本领　角半宽度 $\Delta\theta = \frac{\Delta\lambda}{d\cos\theta_k N}$ = 条纹角间距 $\delta\theta = \frac{k\Delta\lambda}{d\cos\theta}$

⟼ $\Delta\lambda_{min} = \frac{\bar{\lambda}}{kN}$　色分辨本领 $\equiv \frac{\bar{\lambda}}{\Delta\lambda_{min}} = kN$

[3] 自由光谱程　$\Delta\lambda_{FSR} = \frac{\lambda}{k}$

Summary 总结

Chapter Ⅲ

Title 光的衍射章末总结 Date /

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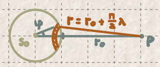
§. Fresnel 衍射

▸【Theorem】Huygens 原理

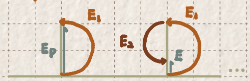


$E(P) = \sum_{n=1}^{m} E_n$

$|E_p| = |E_1| - |E_2| + |E_3| - \cdots \pm |E_m|$

$\begin{cases} m=\text{奇} & |E_p| = \frac{|E_1|}{2} + \frac{|E_m|}{2} \\ m=\text{偶} & |E_p| = \frac{|E_1|}{2} - \frac{|E_m|}{2} \end{cases}$

【Prop】振幅矢量图



Fresnel 圆孔/圆屏衍射

$|E_p| = \frac{|E_1|}{2} \pm \frac{|E_n|}{2}$

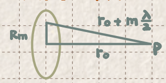
$R^2 = n r_0 \lambda$

$|E_p| = \frac{|E_n|}{2}$

$R^2 = n r_0 \lambda$

泊松亮斑

Fresnel 波带板

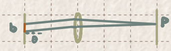


$R_m^2 = m r_0 \lambda$

定义波带板 $f = r_0 = \frac{R_m^2}{m\lambda} \propto \frac{1}{\lambda}$ 次焦距: $\frac{f}{3}$ $\frac{f}{5}$ 等

§. Fraunhofer 衍射

单缝衍射

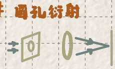


$I_p = I_0 \frac{\sin^2\beta}{\beta^2}$ 其中 $\beta = \frac{\delta_{AB}}{2} = \frac{\pi b \sin\theta}{\lambda}$

$\beta = 0 \mapsto$ 中央主极大 $\beta = \tan\beta \longrightarrow$ 次极大 $\beta = k\pi \longrightarrow$ 极小

宽度 $\pi = \frac{\pi b \theta}{\lambda} \mapsto \theta = \frac{\lambda}{b}$ $d = 2f\theta$

圆孔衍射



$I_p = I_0 \left(\frac{2J_1(\beta)}{\beta}\right)^2$

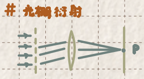
Airy 斑: 第一主极大光斑大小 $\theta_{min} = 1.22\frac{\lambda}{D}$ $L_{min} = 1.22\frac{f\lambda}{D}$

▸【Theorem】Reyleigh 判据 两点像最小分辨距: Airy 斑半径 分辨本领 $\equiv \frac{1}{\theta_{min}} \propto D$

显微镜分辨本领 $y_{min} = \frac{0.61\lambda}{n\sin u}$ n, u: 物方 $n\sin u \equiv NA$ 数值孔径

§. 光栅衍射

光栅衍射



光栅周期: d 缝宽: b 狭缝数: N

$I_p = I_0 \frac{\sin^2\beta}{\beta^2} \frac{\sin^2 N\gamma}{\sin^2\gamma}$ 单缝衍射因子: $\frac{\sin^2\beta}{\beta^2}$ 多光束干涉因子: $\frac{\sin^2 N\gamma}{\sin^2\gamma}$

其中 $\delta = \frac{2\pi}{\lambda} d\sin\theta$ $\beta = \frac{\pi b \sin\theta}{\lambda}$ $\gamma = \frac{\delta}{2} = \frac{\pi d \sin\theta}{\lambda}$

$\gamma = k\pi \longrightarrow$ 主极大 即 $d\sin\theta = k\lambda$ $\gamma = \frac{m}{N}\pi$ $m = 1, 2 \cdots N-1 \longrightarrow$ 主极小

两个主极间有 N-1 个主极小, N 个次极大 主极大时 $\frac{\sin^2 N\gamma}{\sin^2\gamma} = N^2$ N↑ I_{max}↑

$\beta = k\pi \longrightarrow$ 主极小 与 干涉主极大重合: 缺级

$\begin{cases} d\sin\theta = k_1\pi \\ b\sin\theta = k_2\pi \end{cases} \mapsto \frac{d}{b} = \frac{\text{整}(k_1)}{\text{整}(k_2)}$ k_1 级主极大缺失

▸【Remark】对 $d\sin\theta = k\lambda$

色散率 $\frac{d\theta}{d\lambda} = \frac{1}{d}\frac{k}{\cos\theta}$ 色分辨本领 $\equiv \frac{\bar{\lambda}}{\Delta\lambda_{min}} = kN$ 光谱自由程: $\Delta\lambda_{FSR} = \frac{\lambda}{k}$

Summary 总结

Title 光的偏振

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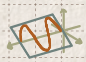
§. 偏振光

▸[Prop] 完全偏振光分为直线偏振光、圆偏振光、椭圆偏振光

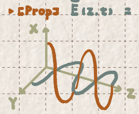
完全非偏振光：自然光　X-Y 振动几率均等

部分偏振光：介于之间



▸[Def] 定义偏振度 $P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$　$0 \le P \le 1$　$P=1$ 完全　$P=0$ 自然

▸[Prop] $\vec{E}(z,t) = \vec{E}_x(z,t) + \vec{E}_y(z,t) = E_x(z,t)\vec{i} + E_y(z,t)\vec{j}$



对单色光简谐振动

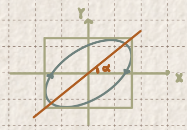
$$\begin{cases} E_x = E_{0x}\cos(\omega t - kz) \\ E_y = E_{0y}\cos(\omega t - kz - \varepsilon) \end{cases}$$

ε: E_x 超前 E_y 的相位差

ε 确定 $\longmapsto$ 完全偏振光　　ε 不确定 $\longmapsto$ 自然光

ε const 时

$$\begin{cases} \left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 - \frac{2E_xE_y}{E_{0x}E_{0y}}\cos\varepsilon = \sin^2\varepsilon \\ \tan 2\alpha = \frac{2E_{0x}E_{0y}\cos\varepsilon}{E_{0x}^2 - E_{0y}^2} \end{cases}$$



$\varepsilon = m\pi$ 时 直线偏振光　$E_x = \pm\frac{E_{0x}}{E_{0y}}E_y$

$\varepsilon = (2m+1)\frac{\pi}{2}$　$E_{0x} = E_{0y}$ 时　圆偏振光

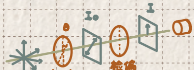
$\varepsilon \in (0,\pi)$ 逆时针　左旋椭圆偏振光　//　$\varepsilon \in (-\pi, 0)$ 顺时针　右旋椭圆偏振光

§. 偏振器

▸[Theorem] Malus 定律　$I(\theta) = I_0\cos^2\theta$

$I(0°) = I_0$

$I(90°) = 0$　消光



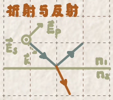
[Remark] 自然光偏振一次 $I_0 = \frac{1}{2}I_自$

§. 偏振途径

二向色性 *

包括 线性导线光栅、人造偏振片、二向色性晶体

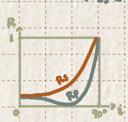
折射与反射



定义 振动分量 S振动　P振动　$\vec{p} = \vec{s} \times \vec{k}$

$$\begin{cases} \text{物质方程} \quad D = \varepsilon E \quad B = \mu H \\ \text{电磁波} \quad \sqrt{\varepsilon}E = \sqrt{\mu}H \quad v = \frac{1}{\sqrt{\varepsilon\mu}} \\ \text{折射定律} \quad n_1\sin i_1 = n_2\sin i_2 \end{cases}$$

$\longmapsto$ S: $\begin{cases} r_s = -\frac{\sin(i_1 - i_2)}{\sin(i_1 + i_2)} \\ t_s = \frac{2\sin i_2\cos i_1}{\sin(i_1 + i_2)} \end{cases}$　P: $\begin{cases} r_p = \frac{\tan(i_1 - i_2)}{\tan(i_1 + i_2)} \\ t_p = \frac{2\sin i_2\cos i_1}{\sin(i_1 + i_2)\cos(i_1 - i_2)} \end{cases}$　$R_\cdot = r_\cdot^2$　$T_\cdot = 1 - R_\cdot$



$i_1 = 0°$　$R_s = R_p = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$

$i_1 + i_2 = 90°$　$R_p = 0$　$i = i_b = \tan^{-1}\left(\frac{n_2}{n_1}\right)$　i_b: Brewster 角

$i_2 = 90°$　$R_s = R_p = 1$

反射起偏：$i = i_b$　$R \sim 15\%$　效果差　S偏振

连续折射起偏：$i = i_b$　20片时　S通过率 $(0.85)^{20} \to 0$　P偏振

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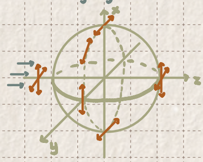
光的散射

▸[Def] Reyleigh 散射：分子散射 $a \ll \lambda$ $I_s \propto \frac{1}{\lambda^4}$ 蓝紫光易散射 红光不易散射

分子在入射光作用下受迫振动 过滤去纵波的部分



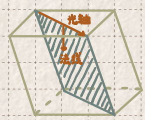
- z方向 自然光
- xoy面 线偏振光
- 其它方向 部分偏振光

双折射

▸[Def] 光入射到各向异性介质的分界面上时，一束折射光不遵从折射定律（e光），另一束为正常折射光（o光） 均为线偏振光

晶体光轴：晶体中的一个方向，光沿该方向传播时 o、e光不分开，传播速度相等

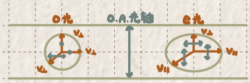
对方解石晶体 光轴 // 钝角角平分线



- 主截面：法线 + 光轴 —— 3个
- 主平面：折射光线 + 光轴 —— 1个

- o 振动方向 ⊥ o 主平面
- e 振动方向 // e 主平面

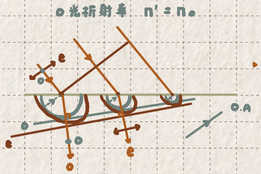
入射面与主截面重合 o振动方向 ⊥ e振动方向



- 振动方向 ⊥ 光轴 $v_\perp$ $v_\perp > v_\parallel$ 正轴晶体 $n_o = \frac{c}{v_\perp}$
- 振动方向 // 光轴 $v_\parallel$ $v_\perp < v_\parallel$ 负轴晶体 $n_e = \frac{c}{v_\parallel}$

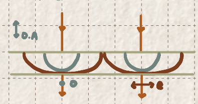
$(n_e - n_o)$ 反映双折射明显程度

o光折射率 $n' = n_o$ e光折射率 $n''^2(\theta) = \frac{n_e^2 n_o^2}{n_o^2 \sin^2\theta + n_e^2 \cos^2\theta}$

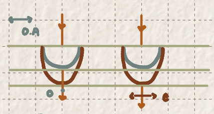


▸[Prop] 考虑正入射：

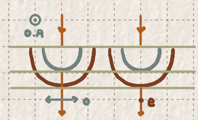
(i) 光轴 ⊥ 表面



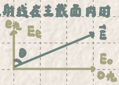
(ii) 光轴 // 表面



(iii) 光轴 // 表面



入射线在主截面内时 o光振动方向 ⊥ e光振动方向



自然光 $I_o = I_e$

线偏振光 $E_o = E\sin\theta$ $E_e = E\cos\theta$

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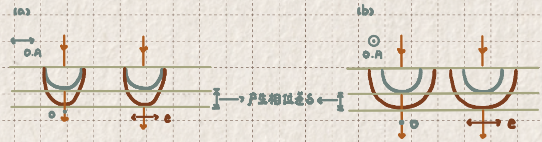
§. 位相延迟器

▸ [Prop] 为获取圆偏振光、椭圆偏振光，引入波片

偏振光类型取决于位相差 ε （如图 ※）



$\delta = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda}(n_o - n_e)d$ d: 晶体厚度

[Def] 定义传播速度快的振动方向为波片快轴，对应快光；反之为慢光

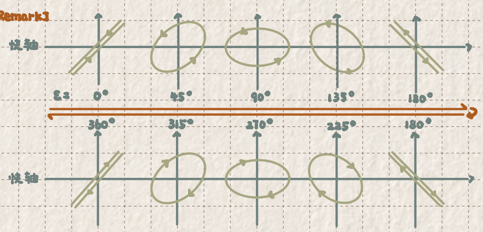
负轴晶体 $v_e > v_o$ e光快光 快轴 // 光轴

[Def] 全波片 $\delta = 2k\pi$ 半波片 $\delta = (2k+1)\pi$ $\frac{1}{4}$ 波片 $\delta = (4k+1)\frac{\pi}{2}$

$d = \frac{k\lambda}{n_o - n_e}$ $d = \frac{1}{n_o - n_e}(2k+1)\frac{\lambda}{2}$ $d = \frac{1}{n_o - n_e}(4k+1)\frac{\lambda}{4}$

不改变偏振状态 角度对称，方向相反 正椭、圆 ⇌ 线

▸ [Remark]



ε: 偏振方向与快轴夹角

[Prop] 实验测定偏振类型

→ 偏振片 → I 不变 → $\frac{1}{4}$ 波片（在偏振片前） — I 不变 → 自然光；I 变化 → 圆偏振

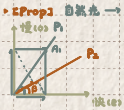
→ 偏振片 → $I_{min} = 0$ → 线偏振

→ 偏振片 → I 变化 → $\frac{1}{4}$ 波片（快轴 // 极大） — I 变化 → 椭圆偏振；I 不变 → 部分偏振

§. 干涉色与旋光性

▸ [Prop] 自然光 → 偏振片 P_1 → 晶体分光 ($e \perp o$) → 偏振片 P_2 → 干涉色

偏振片 P_1 ↓ A_1, I_0；偏振片 P_2 ↓ A_2, I



$I = \underbrace{I_0\cos^2(\alpha-\beta)}_{\text{背景项}} - \underbrace{I_0 \sin 2\alpha \sin 2\beta \sin^2\frac{\delta}{2}}_{\text{干涉项}}$ $\delta = \frac{2\pi}{\lambda}|n_o - n_e|d$

$$\begin{cases} I_{P_1 /\!/ P_2} = I_0 - I_0 \sin^2 2\alpha \sin^2\frac{\delta}{2} \\ I_{P_1 \perp P_2} = I_0 \sin^2 2\alpha \sin^2\frac{\delta}{2} \end{cases}$$

▸ [Prop] 旋光公式 $\theta = \alpha \cdot d$ α: 旋光系数（与波长、材质、浓度有关）

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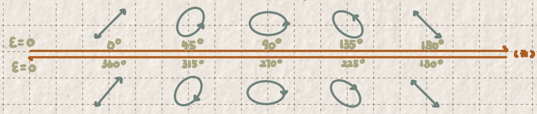
§. 偏振原理

▸ [Def] 自然光、部分偏振光、线 / 圆 / 椭圆偏振光　$P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$

[Prop] $\begin{cases} E_x = E_{0x} \cos(\omega t - kz) \\ E_y = E_{0y} \cos(\omega t - kz + \varepsilon) \end{cases}$　$\varepsilon \longrightarrow$ 偏振原因

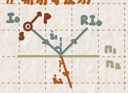
ε = const $\longrightarrow$ 偏振

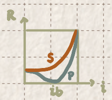


§. 偏振途径

▸ [Theorem] Malus 定律　对偏振光 $I(\theta) = I_0 \cos^2\theta$　对自然光偏振一次 $I_1 = \frac{1}{2} I_0$

折射与反射

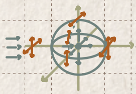




i_b: Brewster 角　此时 $i_1 + i_2 = 90°$　$i_b = \tan^{-1}\left(\frac{n_2}{n_1}\right)$

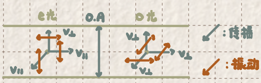
{ 反射起偏　$i = i_b$ 时　反射光仅 S 光　效果差
{ 连续折射起偏　$i = i_b$ 时　折射光 S 光损失 R　$\longmapsto (1-R)^n \xrightarrow{n大} 0$

光的散射

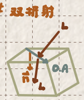


原理：光是横波，振动方向 ⊥ 传播方向

散射光振动方向 ⊥ 散射光入射方向　且 ⊥ 入射光传播方向



双折射



光轴：o.e 光沿该方向传播时不分开

主截面：光轴 + 表面法线确定

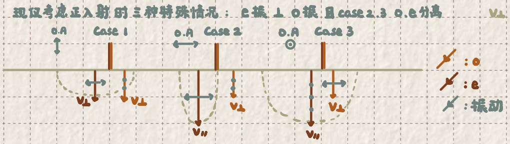
主平面：光轴 + 折射光线确定

{ o 振动方向 ⊥ o 主平面
{ e 振动方向 // e 主平面

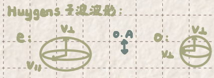
定义 振动方向 ⊥ 光轴有 $V_\perp$　$n_o \equiv \frac{c}{V_\perp}$

// 光轴有 $V_\parallel$　$n_e \equiv \frac{c}{V_\parallel}$

现仅考虑正入射时三种特殊情况：e 振 ⊥ o 振 且 case 2.3 o.e 分离　$V_\perp > V_\parallel$ 正轴　$V_\perp < V_\parallel$ 负轴



Huygens 子波波形：



§. 相位延迟

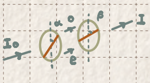
▸ [Prop] In case 2.3　$V_e \neq V_o$　e 振 ⊥ o 振　出射后 $\delta = \frac{2\pi}{\lambda} |n_o - n_e| d$　控制 $|n_o - n_e| d \longrightarrow$ 控制 δ

定义：V 大的为快光　振动方向为快轴

对应 (※) 图 $\rightleftharpoons$ 代表快轴方向　全波片：$\delta = 2k\pi$　半波片：$\delta = (2k+1)\pi$　跳 4 格　$\frac{1}{4}$ 波片：$\delta = (4k+1)\frac{\pi}{2}$　跳 2 格

▸ [Prop] 干涉色　再次偏振 s.t. o 光振动 // e 光振动　$\delta = \frac{2\pi}{\lambda} |n_o - n_e| d$

$I = I_0 \cos^2(\alpha - \beta) - I_0 \sin 2\alpha \sin 2\beta \sin^2 \frac{\delta}{2}$



▸ [Prop] 旋光　$\theta = \alpha \cdot d$　$\alpha = \alpha(\lambda, \rho, n)$

Summary 总结

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附录：光学记背公式

Chpt.1 几何光学

最小偏向角求折射率 $i_1 = \frac{1}{2}(\delta_{min} + \alpha)$ $n = \frac{\sin\frac{\alpha+\delta_{min}}{2}}{\sin\frac{\alpha}{2}}$

(i) 含 r： $\frac{n'}{s'} - \frac{n}{s} = \frac{n'-n}{r}$ $f = \frac{n}{n-n'} r$ $f' = \frac{n'}{n'-n} r$ // 放大率 $\beta = \frac{s'}{n'}\frac{n}{s}$ $\alpha = -\frac{x'}{x}$ $\gamma = \frac{s}{s'}$ $\beta = \alpha\gamma$

(ii) 不含 r： $\frac{f}{f'} = -\frac{n}{n'}$ $\frac{f}{s} + \frac{f'}{s'} = 1$ $xx' = ff'$ 放大镜 $M = \frac{25cm}{f'}$ 显微镜 $M = -\frac{\Delta}{f_1'}\frac{25cm}{f_2'}$

基点法合并 $d = \Delta - f_1' + f_2$ $L_{HH_1} = f_1\frac{d}{\Delta}$ $L_{H'H_2'} = f_2'\frac{d}{\Delta}$ $f = \frac{f_1 f_2}{\Delta}$ $f' = -\frac{f_1' f_2'}{\Delta}$ $S_N = S_N' = f + f'$

Chpt.2 光的干涉

干涉定义 $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$ $\delta = \frac{2\pi}{\lambda}\Delta$ $V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$ $\Delta = k\lambda$ 极大 $\Delta = (k+\frac{1}{2})\lambda$ 极小

I. 双缝干涉 $\Delta = \frac{nd}{L}x$ II. 薄膜干涉 $\Delta_r = 2nt\cos\theta - \frac{\lambda}{2}$ III. 楔形膜 $\Delta = 2\alpha x\cos\theta - \frac{\lambda}{2}$ IV. 牛顿环 $\Delta_r = \frac{r^2}{R} - \frac{\lambda}{2}$

Fabry-Perot 干涉 $I_t = \frac{I_0}{1 + F\sin^2\frac{\delta}{2}}$ $\Delta_t = 2\pi t\cos\theta$ $F = (\frac{2r}{1-r^2})^2$ 条纹精细度 $\mathcal{F} = \frac{\pi}{2}\sqrt{F}$ $\Delta\lambda_{FSR} = \frac{\lambda^2}{2t}$ $\frac{\bar{\lambda}}{\Delta\lambda_{min}} = k\mathcal{F}$

Chpt.3 光的衍射

Fresnel 衍射 $R^2 = n r_0 \lambda$ $R_m^2 = m r_0 \lambda$ $\Longrightarrow$ $f = r_0$ 次焦点 $f' = \frac{1}{3}f$ / $\frac{1}{5}f$ …

Fraunhofer 衍射 (i) 单缝衍射 $I_p = I_0\frac{\sin^2\beta}{\beta^2}$ $\beta = \frac{\pi b\sin\theta}{\lambda}$ $\beta = 0$ 主极大 $\beta = k\pi$ 主极小

(ii) 圆孔衍射 Airy 斑 $\theta_{min} = 1.22\frac{\lambda}{D}$ $L_{min} = f\theta_{min}$ 显微镜 $y_{min} = \frac{0.61\lambda}{n\sin u}$

光栅衍射 $I_p = I_0\frac{\sin^2\beta}{\beta^2}\frac{\sin^2 N\gamma}{\sin^2\gamma}$ $\beta = \frac{\pi b\sin\theta}{\lambda}$ $\gamma = \frac{\pi d\sin\theta}{\lambda}$ $\gamma = k\pi$ 主极大 $\gamma = \frac{m}{N}\pi$ 主极小 $d\sin\theta = k\lambda$

缺极 $\beta = k_1\pi$ $\gamma = k_2\pi$ // 色散率 $\frac{d\theta}{d\lambda}$ $\frac{\bar{\lambda}}{\Delta\lambda_{min}} = kN$ $\Delta\lambda_{FSR} = \frac{\lambda}{k}$

Chpt.4 光的偏振

Malus 定律 $I(\theta) = I_0\cos^2\theta$ Brewster 角 $i_b = \tan^{-1}(\frac{n_2}{n_1})$ 起偏决定 $\varepsilon = \delta = \frac{2\pi}{\lambda}|n_o - n_e|d$

干涉色 $I = I_0\cos(\alpha-\beta) - I_0\sin2\alpha\sin2\beta\sin^2\frac{\delta}{2}$