1	2	3	4	5	6	Sum
/20	/20	/20	/20	/20	/20	/100

Name ______________________ Dec. 29, 2005 Thursday

Department _________________

SID _______________________

University Physics

The Solution for the Practice Final Exam

School of Intensive Instruction in Sciences and Arts, Nanjing University

Physical Constants

Electron charge magnitude	$e = 1.60\times10^{-19}$ C
Dielectric constant of vacuum	$\varepsilon_0 = 8.85\times10^{-12}$ F/m
Permeability of vacuum	$\mu_0 = 4\pi\times10^{-7}$ N/A^2
Vacuum speed of light	$c = 3.00\times10^{8}$ m/s
Electron rest mass	$m_e = 9.11\times10^{-31}$ kg $= 0.511$Mev/c^2
Proton rest mass	$m_n = 1.672\times10^{-27}$ kg $= 938.3$MeV/c^2
Neutron rest mass	$m_n = 1.675\times10^{-27}$ kg $= 939.6$MeV/c^2
Atomic mass unit	1 amu $= 1.660\times10^{-27}$ kg $= 931.5$MeV/c^2
Electron volt (eV)	1eV$=1.60\times10^{-19}$ J
Planck constant	$h = 6.63\times10^{-31}$ Js $\hbar = 2.11\times10^{-31}$ Js $= 0.66\times10^{-15}$ eVs
Boltzmann constant	$k_B = 1.38\times10^{-23}$ J/K $= 8.62\times10^{-5}$ eV/K
Bohr radius	$a_0 = 0.529$ Angstroms

Select five out of following six problems.

1. (20pts). (a) Show that the fractional loss of energy of a photon during a Compton collision is given by

$$\frac{\Delta E}{E} = \frac{h\nu'}{m_e c^2}(1-\cos\phi)$$

where ν' is the frequency of scattered photon, ϕ is the angle between the momentum of incident photon and the momentum of scattered photon..

(b) Show that when a photon of frequency ν scatters from a free (rest) electron, the maximum kinetic energy of the electron is given by

$$K_{\max} = \frac{2(h\nu)^2}{2h\nu + m_e c^2}.$$

Proof: (a) The Compton shift is given by

$$\lambda' - \lambda = \frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{m_e c}(1-\cos\phi)$$

or

$$\frac{\nu - \nu'}{\nu} = \frac{h\nu'}{m_e c^2}(1-\cos\phi).$$

Using the de Broglie relation $E = h\nu$, we have

$$\frac{E-E'}{E}=\frac{h\nu'}{m_e c^2}(1-\cos\phi) \quad \text{or} \quad \frac{\Delta E}{E}=\frac{h\nu'}{m_e c^2}(1-\cos\phi).$$

(b) The kinetic energy of the scattered electron is

$$K=h\nu-h\nu'=hc\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right).$$

From the expression of the Compton wavelength shift, we can see that the maximum wavelength of the scattered photon can be obtained when $\cos\phi=-1$, that is

$$\lambda'_{\max}=\lambda+\frac{2h}{m_e c}.$$

Then, the maximum kinetic energy of the electron is

$$K_{\max}=hc\left(\frac{1}{\lambda}-\frac{1}{\lambda'_{\max}}\right)=\frac{hc(\lambda'_{\max}-\lambda)}{\lambda\lambda'_{\max}}=\frac{\frac{hc}{\lambda}\cdot\frac{2h}{m_e c}}{\lambda+\frac{2h}{m_e c}}$$

$$=\frac{h\nu\cdot\frac{2hc}{\lambda}}{m_e c^2+\frac{2hc}{\lambda}}=\frac{2(h\nu)^2}{2h\nu+m_e c^2}.$$

2.(20 pts) An electron is moving on a circular mesoscopic ring of perimeter l.

(a) Write the Schrödinger's equation of the electron and the boundary condition for the wave function of the electron.

(b) Obtain the energy levels and the corresponding stationary wave functions of the electron.

(c) Find the degeneracy of the energy levels.

Solution: (a) The Schrödinger Equation is

$$-\frac{\hbar^2}{2m_e}\frac{d^2\psi(x)}{dx^2}=E\psi(x)$$

and boundary condition is $\psi(x+l)=\psi(x)$.

(b) The Schrödinger equation can be rewritten as

$$\frac{d^2\psi(x)}{dx^2}+k^2\psi(x)=0, \quad k^2=\frac{2mE}{\hbar^2}.$$

The solution is then

$$\psi(x)=Ae^{ikx}.$$

By using the boundary condition, nwe have

$$Ae^{ik(x+l)}=Ae^{ikx} \quad \text{or} \quad e^{ikl}=1.$$

Thus

$$kl=2\pi n \ , \ n=0,\ \pm1,\ \pm2,\ \pm3,\cdots$$

Therefore $k_n=\dfrac{2\pi n}{l}$, $E_n=\dfrac{n^2\pi^2h^2}{2m_e l^2}$ and $\psi_n(x)=Ae^{i\frac{2\pi nx}{l}}$.

(c) One can easily see that the degree of degeneracy for E_0 is 1 and the degrees of degeneracy for all other energy levels are 2.

3.(20pts) The asymmetry term of the empirical nuclear binding energy can be obtained through the Fermi gas model of the nucleus that is analogous to electron gas model of the metal. In the model it assumed that the nucleus is consisted of independent proton and neutron gases. Consider a nucleus consisting of N neutrons and Z protons respectively, where $N+Z=A$, the mass number of the nucleus. The radius of the nucleus is given by $R=r_0A^{1/3}$ and $r_0=1.2$ fm.

(a) Derive the expressions of Fermi energies for both neutron and proton gases.
(b) Find the total kinetic energy of the nucleons when the system is in its ground state.
(c) Show that the kinetic energy is minimized when $Z = N = A/2$.
(d) Find the difference of the kinetic energy from its minimum value when the proton number is deviated from $A/2$.
Solution: (a)

$$E_F^p = \frac{\hbar^2}{2m_p}\left(3\pi^2\frac{Z}{V}\right)^{2/3}, \quad E_F^n = \frac{\hbar^2}{2m_n}\left(3\pi^2\frac{N}{V}\right)^{2/3}$$

where $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_0^3 A$. Then

$$E_F^p = \frac{\hbar^2}{2m_p}\left(\frac{9\pi}{4r_0^3}\right)^{2/3}\left(\frac{Z}{A}\right)^{2/3}, \quad E_F^n = \frac{\hbar^2}{2m_n}\left(\frac{9\pi}{4r_0^3}\right)^{2/3}\left(\frac{N}{A}\right)^{2/3}.$$

(b) The total kinetic energy of the nucleons is

$$U = \frac{3}{5}(ZE_F^p + NE_F^n) = C\frac{(Z^{5/3}+N^{5/3})}{A^{2/3}}$$

where $C = \frac{9\hbar^2}{20m_p r_0^2}\left(\frac{3\pi^2}{2}\right)^{1/3}$.

(c) Let $D = N - Z$. Since $A = N + Z$, then we have

$$N = \frac{1}{2}(A+D) = \frac{A}{2}\left(1+\frac{D}{A}\right), \qquad Z = \frac{1}{2}(A-D) = \frac{A}{2}\left(1-\frac{D}{A}\right).$$

Substituting into the expression of the total kinetic energy, we obtain

$$U = \frac{CA}{2^{5/3}}\left[\left(1+\frac{D}{A}\right)^{5/3} + \left(1-\frac{D}{A}\right)^{5/3}\right].$$

Using

$$\left(1\pm\frac{D}{A}\right)^{5/3} \approx 1 \pm \frac{5}{3}\frac{D}{A} + \frac{5}{9}\left(\frac{D}{A}\right)^2$$

We finally obtain

$$U = \frac{CA}{2^{2/3}} + \frac{5C}{9\cdot 2^{2/3}A}D^2 = \frac{CA}{2^{2/3}} + \frac{5C}{9\cdot 2^{2/3}A}(N-Z)^2.$$

One can easily see that the kinetic energy is minimized when $Z = N = A/2$.
(d)

$$U_{\min} = \frac{CA}{2^{2/3}}, \qquad U - U_{\min} = \frac{5C}{9\cdot 2^{2/3}A}(N-Z)^2.$$

4.(20pts) The half-life of ^{235}U is $7.04\times 10^8\, y$.
(a) What is the decay constant of ^{235}U?
(b) A sample of rock, which solidified with the earth 4.55×10^9 years ago, contains N atoms of ^{235}U. How many ^{235}U atoms did the same rock have at the time it solidified?

Solution: (a) The decay constant is

$$\lambda = \frac{\ln 2}{t_{1/2}} = 0.98\times 10^{-9}\, y^{-1}$$

(b) $$N = N_0 e^{-\lambda t}, \quad N_0 = Ne^{\lambda t} = N\exp(0.98\times 10^{-9}\times 4.55\times 10^9) = 86.4N.$$

5. The quark combinations of following particles are

$$K^0 - d\bar{s} \qquad \pi^- - d\bar{u} \qquad K^- - s\bar{u} \qquad p - uud \qquad \Lambda^0 - uds \qquad \Sigma^- - dds.$$

(a) Find the charge number Q, the baryon number B and the strangeness S of these particles.
(b) Check the conservations of baryon number, charge and strangeness in the following processes.
(c) Determine if these processes are mediated by the strong or weak interaction, or are forbidden. Explain why.

(1) $\Lambda^0 \to \pi^- + p^+$

(2) $\pi^- + p^+ \to \Lambda^0 + K^0$

(3) $K^- + p \to \Sigma^- + \pi^+$

(4) $K^- + p \to \Lambda^0 + K^0$

Quantum numbers of three quarks

Quark symbol	Charge number	Spin	Baryon number	Strangeness
u	2/3	1/2	1/3	0
d	−1/3	1/2	1/3	0
s	−1/3	1/2	1/3	−1

Answer: (a)

Particle Symbol	Q	B	S
K^0	0	0	1
π^-	-1	0	0
K^-	-1	0	-1
p	1	1	0
Λ^0	0	1	-1
Σ^-	-1	1	-1

(b) & (c)

(1)
$$\Lambda^0 \to \pi^- + p^+$$
Q $0 = -1 + 1$
B $1 = 0 + 1$
S $-1 \neq 0 + 0$

Since the strangeness is not conserved, the process is a weak decay.

(2)
$$\pi^- + p^+ \to \Lambda^0 + K^0$$
Q $-1+1 = 0 + 0$
B $0+1 = 1 + 0$
S $0 + 0 = -1+1$

The process is due to the strong interaction (collision between hadrons).

(3)
$$K^- + p \to \Sigma^- + \pi^+$$
Q $-1 + 1 = -1 + 1$
B $0 + 1 = 1 + 0$
S $-1 + 0 = -1 + 0$

The process is due to the strong interaction (collision between hadrons).

(4)
$$K^- + p \to \Lambda^0 + K^0$$
Q $-1+1 = 0 + 0$
B $0 + 1 = 1 + 0$
S $-1+0 \neq -1 + 1$

It should be the strong interaction (collision between hadrons). But the strangeness is not conserved. So the process is forbidden.

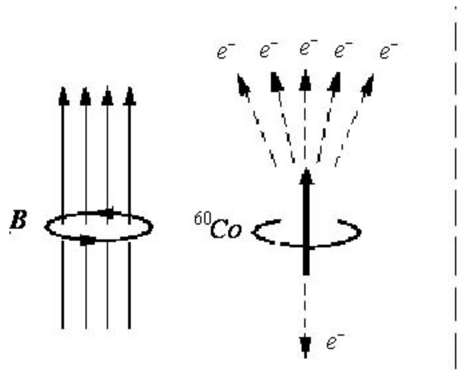
6.(20pts) In the ^{60}Co beta-decay experiment, the electrons emitted in the direction parallel to the applied magnetic field are found to be more than those emitted in the direction anti-parallel to the field. The original process is shown in the figure.

(a) Draw a figure to show the mirror image process of the beta-decay.

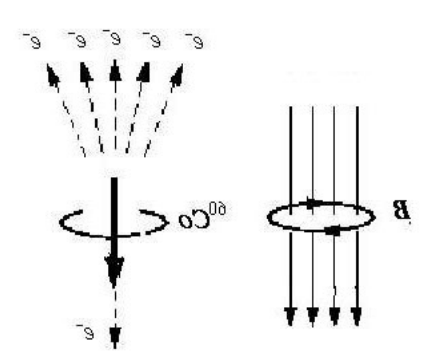
(b) Can the mirror image process occur in nature? Why?

(c) What conclusion can you draw if the mirror process can/cannot occur in nature?

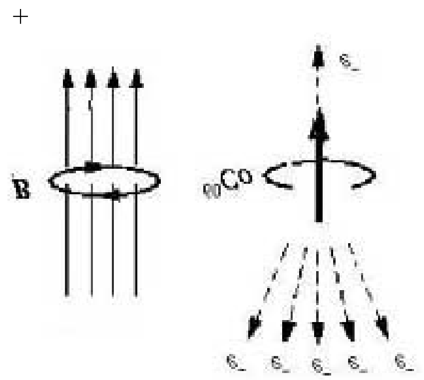
Solution: (a)



Original process



Mirror image process



(b) The mirror image process cannot occur in nature because the mirror image is just the process in contrast to the experimental result.

(c) The mirror image process cannot occur in nature. Therefore, in the process the parity is not conserved.