1	2	3	4	5	6	Optional	Sum
/20	/20	/20	/20	/20	/20	/20	/100

Name ______________________ Dec. 24, 2003 Wednesday

Department _________________

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University Physics

Sample Problems for Final Examination

School of Intensive Instruction in Sciences and Arts, Nanjing University

Physical Constants

Electron charge magnitude	$e = 1.60\times10^{-19}$ C
Dielectric constant of vacuum	$\varepsilon_0 = 8.85\times10^{-12}$ F/m
Permeability of vacuum	$\mu_0 = 4\pi\times10^{-7}$ N/A^2
Vacuum speed of light	$c = 3.00\times10^{8}$ m/s
Electron rest mass	$m_e = 9.11\times10^{-31}$ kg $= 0.511$MeV/c^2
Proton rest mass	$m_p = 1.672\times10^{-27}$ kg $= 938.3$MeV/c^2
Neutron rest mass	$m_n = 1.675\times10^{-27}$ kg $= 939.6$MeV/c^2
Atomic mass unit	1 amu $= 1.660\times10^{-27}$ kg $= 931.5$MeV/c^2
Electron volt (eV)	1eV$=1.60\times10^{-19}$ J
Planck constant	$h = 6.63\times10^{-31}$ Js $\hbar = 2.11\times10^{-31}$ Js $= 0.66\times10^{-15}$ eVs
Boltzmann constant	$k_B = 1.38\times10^{-23}$ J/K $= 8.62\times10^{-5}$ eV/K
Bohr radius	$a_0 = 0.259$ Angstroms

Select five out of following six problems.

1. (20pts). (a) In Compton scattering, find the maximum increase of the wavelength of the photon.
(b) If the wavelength of the scattered photon is the twice of the wavelength of the incident photon, what is the minimum energy of the incident X-ray?

Solution: (a) The Compton shift is

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1-\cos\phi)$$

where ϕ is the angle between the momentum of incident photon and the momentum of scattered photon. One can easily see that when $\cos\phi = -1$ or $\phi = \pi$, the increase of the wavelength is maximum:

$$\Delta\lambda_{\max} = \frac{2h}{m_e c} = 2\Lambda = 4.852\times10^{-12}\,m \quad (\Lambda \text{ is the Compton wavelength})$$

(b) $\lambda' = 2\lambda$ or $\Delta\lambda = \lambda$

$$\lambda = \frac{h}{m_e c}(1-\cos\phi) = \Lambda(1-\cos\phi)$$

When $\cos\phi = -1$ or $\phi = \pi$, we have the maximum wavelength $\lambda_{\max} = 2\Lambda$ and the minimum energy of incident photon

$$E_{\min} = \frac{hc}{\lambda_{\max}} = \frac{hc}{2}\frac{m_e c}{h} = \frac{m_e c^2}{2} = 0.26MeV$$

2.(20 pts) (a) Write the time-dependent Schrödinger equation and the stationary (time independent) Schrödinger equation for a particle of mass m moving in the one-dimensional harmonic potential

$$U(x) = \frac{1}{2}kx^2$$

(b) The solution for the stationary Schrödinger equation in the one-dimensional harmonic potential is

$$E_n = (n + \tfrac{1}{2})\hbar\omega,\ \psi_n(x) = N_n e^{-\frac{\alpha^2 x^2}{2}} H_n(\alpha x),\ n = 1,\ 2,\ 3,\ \ldots$$

where $\omega = \sqrt{k/m}$, $\alpha = \sqrt{\mu\omega/\hbar}$, $N_n = \left(\frac{\alpha}{\sqrt{\pi}2^n n!}\right)^{\frac{1}{2}}$ and $H_n(\xi)$ is the Hermite polynomial with the highest power n. We know that $H_n(\xi)$ is an even or odd function if n is even or odd. If there is a particle of mass m moving in the potential

$$U(x) = \begin{cases} \frac{1}{2}kx^2, & x \ge 0, \\ +\infty, & x < 0, \end{cases}$$

find the stationary wave functions and the energy levels of the particle.

Solution: (a) The time-dependent Schrödinger equation is

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2\right)\psi(x,t),$$

and the stationary Schrödinger equation is

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + \frac{1}{2}kx^2\psi(x) = E\psi(x).$$

(b) Since the potential is infinity at $x < 0$, the wave function must be zero for $x < 0$. Due to the continuity of the wave function, the boundary condition for the wave function at $x = 0$ is $\psi(x = 0) = 0$. Then the eigenwavefunction must be odd in order to fulfill the boundary condition. Therefore, the energy levels and corresponding stationary wavefunctions are

$$E_n = (n + \tfrac{1}{2})\hbar\omega,\ \psi_n(x) = \begin{cases} \sqrt{2}N_n e^{-\frac{\alpha^2 x^2}{2}} H_n(\alpha x), & x \ge 0, \\ 0, & x < 0, \end{cases},\quad n = 1,\ 3,\ 5, \ldots$$

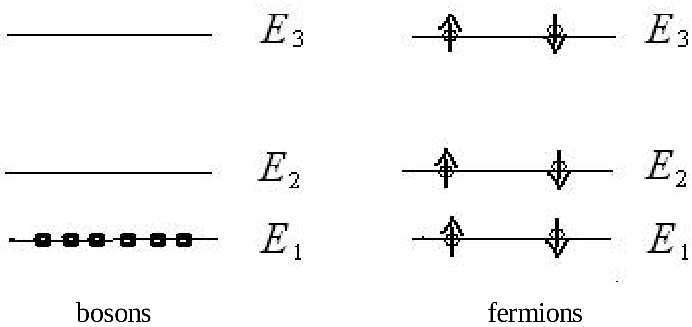
3.(20pts) If we put 6 identical bosons or 6 identical spin-1/2 fermions into a one-dimensional infinite well of width a (the energy levels for a particle of mass m in the well is $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}, n = 1,2,3\cdots$).

(a) Draw a schematic diagram to show how the energy levels are filled in each case when the system is in the ground state.

(b) Find the ground state energy of the system in each case.

(c) The atomic mass of a copper atom is 63.55; the density is 8.96gcm^{-3}. Calculate the Fermi energy E_F of the electron gas in copper. (Assume there is 1 valence electron per atom for copper.)

Solution: (a)



(b) The ground state energy for the boson system is $E = 6E_1 = \frac{3\pi^2\hbar^2}{ma^2}$. And the ground state energy for the fermion system is $E = 2(E_1 + E_2 + E_3) = \frac{14\pi^2\hbar^2}{ma^2}$.

(c) In the k-space (wave vector space), a volume of $(2\pi)^3/V$ corresponds to a quantum state. Therefore, we have

$$\frac{2V \times \frac{4}{3}\pi k_F^3}{(2\pi)^3} = N \quad \text{or} \quad k_F = \left(3\pi^2 \frac{N}{V}\right)^{1/3}.$$

Therefore

$$E_F = \frac{\hbar^2 k_F^2}{2m_e} = \frac{\hbar^2}{2m_e}\left(3\pi^2 \frac{N}{V}\right)^{2/3}.$$

Since $N/V = \frac{8.96\times10^6}{63.55}\times 6.023\times10^{23} = 8.49\times10^{28}(\text{m}^{-3})$,

$$E_F = \frac{\hbar^2}{2m_e}\left(3\pi^2 \frac{N}{V}\right)^{2/3} = \frac{(0.66\times10^{-15}\,\text{eVs})^2}{2\times0.511\times10^6\,\text{eV}/\left(3.0\times10^8\,\text{ms}^{-1}\right)^2}\left(3\pi^2\times8.49\times10^{28}\,\text{m}^{-3}\right)^{2/3}$$

$$= 7.04\text{eV}$$

4. (20pts) Radioactive nuclei with decay constant λ are produced in an accelerator at a constant rate R_p.
(a) Derive the differential equation for the number of radioactive nuclei N
(b) If the number of the nuclei is zero at $t = 0$, find the number N as a function of time t.
(c) ^{62}Cu is produced at a rate 100 per second by placing ordinary copper (^{63}Cu) in a beam of high-energy protons and ^{62}Cu decays by beta-decay with a half-life of 10 min. How many ^{62}Cu are there after the time long enough so that $dN/dt \to 0$?

Solution: (a) The differential equation for the nuclei number is

$$dN/dt = R_p - \lambda N.$$

(b) The equation can be rewritten as

$$\frac{d(R_p - \lambda N)}{R_p - \lambda N} = -\lambda dt.$$

Integrating both sides of above equation we have

$$\ln(R_p - \lambda N) = -\lambda t + C'$$

or $R_p - \lambda N = Ce^{-\lambda t}$. Substituting $N(t=0) = 0$ into the equation, we obtain $C = R_p$. Then

$$N = \frac{R_p}{\lambda}(1 - e^{-\lambda t}).$$

(c) When $t \to +\infty$, $N \to R_p/\lambda$ and $dN/dt \to 0$. Then, when $t \to +\infty$,

$$N = R_p/\lambda = \frac{R_p t_{1/2}}{\ln 2} = \frac{100\times600}{\ln 2} = 86562.$$

5.(20pts) The empirical binding energy of a nucleus can be written as

$$E_b = a_1 A - a_2 A^{2/3} - a_3 Z^2 A^{-1/3} - a_4 (N - Z)^2 A^{-1}$$

(the pairing energy is not included), where Z is the atomic number, A is the mass number and N is the neutron number of the nucleus. The values of the constants are

$$a_1 = 15.8MeV,\ a_2 = 17.8MeV,\ a_3 = 0.71MeV,\ a_4 = 23.7MeV.$$

(a) Determine the atomic number of the most stable nucleus for a given mass number A.
(b) Explain the physics meaning of the result.

Solution: Using $N+Z=A$ or $N=A-Z$, the binding energy can be rewritten as

$$E_b = a_1 A - a_2 A^{2/3} - a_3 Z^2 A^{-1/3} - a_4 (A-2Z)^2 A^{-1}.$$

The most stable nucleus is the nucleus with maximum binding energy. Thus, from

$$\frac{\partial E_b}{\partial Z} = -2a_3 Z A^{-1/3} + 4a_4 (A-2Z) A^{-1} = 0,$$

we obtain

$$Z = \frac{A}{2\left(1+\frac{a_3}{4a_4}A^{2/3}\right)} = \frac{A}{2+0.015A^{2/3}}$$

(b) 1. When the mass number is small, the atomic number is approximately the half of the mass number. That means the numbers of neutrons and protons are almost the same for the light nuclei. This is due to the asymmetry term of binding energy (the fourth term). The magnitude of this term is the smallest (zero) when $Z=N$.

2. As the atomic number increases, the Coulomb repulsion energy which is proportional to the square of atomic number increases rapidly. Then the ratio of the neutrons must be increased to dilute the Coulomb repulsion.

6. (20pts) Determine if the following processes are mediated mainly by strong, electromagnetic or weak interactions or forbidden. Explain why.

(a) $\eta \to \pi^+ + \pi^- + \pi^0$.

(b) $\rho \to \pi^+ + \pi^-$

(c) $\pi^+ \to \mu^+ + \nu_e$

(d) $K^0 \to \pi^+ + \pi^-$

not required!!!

Answer:(a)The process is mainly due to the electromagnetic interaction because the lifetime of η is shorter than weak interaction lifetimes. (Note: This problem is too difficult at present level of the course. Problems like this will not appear in the final exam.)

(b) It is mainly due to the strong interaction.

(c) It is forbidden because it violates the lepton number conservation.

(d) The decay is mediated mainly by weak interaction because in the process the strangeness does not conserve.

Optional Problem (20pts) Consider a spinless positive pion decaying at rest. (The initial angular momentum and linear momentum of the pion are zero.) A positive muon and a muon neutrino are emitted in opposite directions with equal and opposite momenta. Meanwhile, the two emitted particles have equal but opposite spins due to the angular momentum conservation.

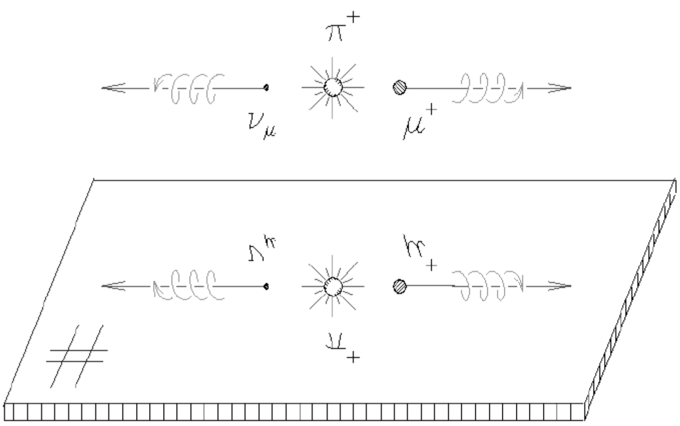
(a) Draw a figure to show the process and the mirror image process of the decay.

(b) Can the mirror image process occur in nature? Why?

(c) What interaction governs the positive pion decay?

(d) What conclusion can you draw if the mirror process can/cannot occur in nature?

Solution: (a)



(b) The mirror image process cannot occur in nature because the there exists no right handed neutrino (neutrino with helicity +1). (A neutrino is always left handed (or has negative helicity)and an anti-neutrino is always right handed (or has positive helicity).)

(c) The decay is governed by the weak interaction.

(d) The mirror image process cannot occur in nature. Therefore, in the process the parity is not conserved.