Galaxy Exam Review

Malin Ouyang
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I. SURFACE BRIGHTNESS

The surface brightness of a galaxy $I(x)$ is the amount of light per square arcsecond on the sky at a particular point x in the image. Consider a small square patch of side D in a galaxy that we view from a distance d, so that it subtends an angle $\alpha = D/d$ on the sky. If the combined luminosity of all the stars in this region is L, its apparent brightness$F$ is given by:

$$F = \frac{L}{4\pi d^2} \tag{1}$$

then the surface brightness is:

$$I(\mathbf{x}) \equiv \frac{F}{\alpha^2} = \frac{L/\left(4\pi d^2\right)}{D^2/d^2} = \frac{L}{4\pi D^2} \tag{2}$$

I is usually given in mag arcsec^{-2} (the apparent magnitude of a star that appears as bright as one square arcsecond of the galaxy's image), or in $L_\odot\ pc^{-2}$. The surface brightness at any point does not depend on distance unless d is so large that the expansion of the Universe has the effect of reducing $I(x)$;Contours of constant surface brightness on a galaxy image are called isophotes. We generally measure surface brightness in a fixed wavelength band, just as for stellar photometry, we generally measure surface brightness in a fixed wavelength band, just as for stellar photometry.

II. THE CLASSIFICATION OF GALAXIES

Based on such features, Hubble ordered galaxies in a morphological sequence, which is now referred to as the Hubble sequence or Hubble tuning-fork diagram (see pdf page 61). Hubble's scheme classifies galaxies into four broad classes:

1. Elliptical galaxies: These have smooth, almost elliptical isophotes and are divided into subtypes E0, E1, ... , E7, where the integer is the one closest to 10(1b/a), with a and b the lengths of the semimajor and semiminor axes

2. Spiral galaxies: These have thin disks with spiral arm structures. They are divided into two branches, **barred spirals** and **normal spirals**, according to **whether or not a recognizable bar-like structure is present in the central part of the galaxy**. On each branch, galaxies are further divided into three classes, a, b and c, according to the following three criteria:

   (a) the fraction of the light in the central bulge

   (b) the tightness with which the spiral arms are wound

   (c) the degree to which the spiral arms are resolved into stars, HII regions and ordered dust lanes

   These three criteria are correlated: spirals with a pronounced bulge component usually also have tightly wound spiral arms with relatively faint HII regions, and are classified as Sa. On the other hand, spirals with weak or absent bulges usually have open arms and bright HII regions and are classified as Sc. When the three criteria give conflicting indications, **Hubble put most emphasis on the openness of the spiral arms.**

3. Lenticular or S0 galaxies: This class is intermediate between ellipticals and spirals. Like ellipticals, lenticulars have a smooth light distribution with no spiral arms or HII regions.Like spirals they have a thin disk and a bulge, but the bulge is more dominant than that in a spiral galaxy. They may also have a central bar, in which case they are classified as SB0.

4. Irregular galaxies: These objects have neither a dominating bulge nor a rotationally symmetric disk and lack any obvious symmetry. Rather, their appearance is generally patchy,dominated by a few HII regions. Hubble did not include this class in his original sequence because he was uncertain whether it should be considered an extension of any of the other classes. Nowadays irregulars are usually included as an extension to the spiral galaxies.

Ellipticals and lenticulars together are often **referred to as early-type galaxies**, while the spirals and irregulars make up th**e class of late-type galaxies**.

III. COMPONENTS OF SPIRAL GALAXY

Our position in the Milky Way's disk gives us a detailed and close-up view of a fairly typical large spiral galaxy. Spiral galaxies are distinguished from S0 systems by the multi-armed spiral pattern in the disk. The disks of spiral galaxies still retain some gas, whereas S0 systems have lost their disk gas, or converted it into stars. Both S0 and spiral galaxies can show a central linear bar.

A. BHs

For the center BH in the MW, we primarily use point source gravitational potential for candidate:

$$\Phi = -\frac{GM}{r} \tag{3}$$

where $M = 4.3 \times 10^6 M_\odot$.

B. NFW profile

For nfw profile,its density:

$$\rho = \frac{\rho_0}{\frac{r}{R_s}(1+\frac{r}{R_s})^2} \tag{4}$$

so we can calculate its gravitational potential along radius:

$$\Phi = -\frac{4\rho_0 {R_s}^3}{r}\ln(1+\frac{r}{R_s}) \tag{5}$$

where we set three parameters in natural units($G = c = 1$):

$$M_{200} = 0.82 \times 10^{12} M_\odot, c = 13.3, r_{200} = 207 \times 1.02938 \times 10^{11} s \tag{6}$$

where c is the halo-mass concentration of GW,thus,we can calculate the R_s and ρ_0:

$$\rho_0 = \frac{M_{200}}{4\pi r_{200}^3}\frac{c^3}{\log(1+c) - c/1+c}, Rs = r_{200}/c \tag{7}$$

C. Bulge of Baryonic model B2

For the bulge, we substitute the original profile used with a Hernquist potential that is given as:

$$\Phi_{Hernquist}(r) = -\frac{GM_{bulge}}{r_b + r}, \tag{8}$$

where we set $M_{bulge} = 1.55 \times 10^{10} M_\odot$ and $r_b = 0.7kpc$.

D. Disk of Bulge of Baryonic model B1

The thin and the thick disks are represented independently by Miyamoto-Nagai potentials that are expressed as:

$$\Phi_{MN}(R, z) = -\frac{GM_{disk}}{\sqrt{R^2 + (R_d + \sqrt{z^2 + z_d^2})^2}} \tag{9}$$

where we set $M_{thin} = M_{thick} = 3.944 \times 10^{10} M_\odot, R_d^{thin} = 5.3kpc, z_d^{thin} = 0.25kpc, R_d^{thick} = 2.6kpc, z_d^{thin} = 0.8kpc$ for the disks.

E. Circumgalactic medium(CGM)

The CGM potential are expressed as:

$$\Phi_{\rm CGM} = -\frac{4\pi(200\rho_{\rm cri}A_{\rm CGM}f_{\rm bar})r^{\beta+2}}{r_{200}^{\beta}}\left(\frac{1}{\beta+2}-\frac{1}{\beta+3}\right)-\frac{4\pi(200\rho_{\rm cri}A_{\rm CGM}f_{\rm bar})}{r_{200}^{\beta}}\left(\frac{(2r_{200})^{\beta+2}}{\beta+2}\right) \tag{10}$$

where $\rho_{\rm cri} = 1.253063 \times 10^{-36} = \frac{3H^2}{8\pi \rm G}, A_{\rm CGM} = 0.190, f_{\rm bar} = 0.157, \beta = -1.46$.

IV. TULLY–FISHER RELATION

Although spiral galaxies show great diversity in luminosity, size,rotation velocity and rotation-curve shape, they obey a well-defined scaling relation **between luminosity L and rotation velocity** (usually taken as the **maximum of the rotation curve** well away from the center, V_{max}).This is known as the Tully–Fisher relation, the Tully–Fisher relation is also important for our understanding of galaxy formation and evolution, as it defines a relation between dynamical mass (due to stars, gas, and dark matter) and luminosity. The observed Tully–Fisher relation is usually expressed in the form $L = AV_{max}^{\alpha}$, where A is the zero-point and α is the slope. The observed value of α is between 2.5 and 4, and is larger in redder bands.This tight relation can be used to estimate the distances to spiral galaxies.

V. LINDBLAD RESONANCE

Lindblad resonances affect stars at such distances from a disc galaxy's centre where the natural frequency of the radial component of a star's orbital velocity is close to the frequency of the gravitational potential maxima encountered during its course through the spiral arms. If a star's orbital speed around the galactic centre is greater than that of the part of the spiral arm through which it is passing, then an inner Lindblad resonance occurs—if smaller, then an outer Lindblad resonance.[4] At an inner resonance, a star's orbital speed is increased, moving the star outwards, and decreased for an outer resonance causing inward movement.

VI. ROTATION CURVE

The predominant motion of gas in a spiral galaxy is rotation; random speeds in the HI gas are typically only$8-10kms^{-1}$, even less than for the stars. we can assume that, at radius R, a gas cloud follows a near-circular path with speed V(R). All we can detect of this motion is the radial velocity Vr toward or away from us; its value at the galaxy's center, V_{sys}, is the systemic velocity. Suppose that we observe a disk in pure circular rotation, tilted at an angle i to face-on, We can specify the position of a star or gas cloud by its radius R and azimuth ϕ,measured in the disk from the diameter AB lying perpendicular to our viewing direction. There, the radial velocity is

$$V_{\rm r}(R,i) = V_{\rm sys} + V(R)\sin i \cos\phi$$

A. spider diagram

Contours of constant V_r connect points with the same value of V(R) cos ϕ, forming a 'spider diagram'. The line AB is the kinematic major axis,the azimuth where Vr deviates furthest from V_{sys}. In the central regions, where approximately $V(R)R$, the contours are parallel to the minor axis; further out, where the rotation speed is nearly constant, they run radially away from the center.

We can find V(R) and the inclination i by choosing values so that the computed velocity contours are close to those measured in HI. Figure 5.20 shows the rotation curve V(R) derived from HI and CO observations of NGC 7331. It climbs steeply over the first 1–2 kpc, then remains approximately flat out to the last measured points, at around 37 kpc. As in many giant spirals, the rising part of the rotation curve is very steep; often, as here, HI observations lack the spatial resolution to follow the rapid climb. At all radii, the angular speed V(R)/R is decreasing; gas further out takes longer to complete an orbit about the galactic center. This differential rotation is typical of spiral galaxies.The rotation curve is a direct measure of the gravitational force within a disk. Assuming, for simplicity, spherical symmetry, the total enclosed mass within radius r can be estimated from:

$$M(r) = rV_{\rm rot}^2(r)/G$$

B. observation detections

Unlike elliptical galaxies which contain gas predominantly in a hot and highly ionized state, the gas component in spiral galaxies is mainly in neutral hydrogen (HI) and molecular hydrogen (H2). Observations in the 21-cm lines of HI and in the mm-lines of CO have produced maps of the distribution of these components in many nearby spirals. **The gas mass fraction increases from about 5% in massive, early-type spirals (Sa/SBa) to as much as 80% in low mass, low surface brightness disk galaxies.** In general, while the distribution of molecular gas typically traces that of the stars, **the distribution of HI is much more extended and can often be traced to several Holmberg radii**. Analysis of **emission from HII regions in spirals provides the primary means for determining their metal abundance** .

1. optical long-slit or IFU spectroscopy of HII region emission lines.

2. radio or millimeter interferometry of line emission from the cold gas such as CO.

3. the HI gas is usually more extended than the ionized gas associated with HII regions, rotation curves can be probed out to larger galactocentric radii using spatially resolved 21-cm observations than using optical emission lines

VII. DOUBLE-HORN PROFILE

If we want to know only the peak rotation speed Vmax in a galaxy, we can use a single-dish radio telescope with a large enough beam to include all the HI gas, and measure how much there is at each velocity. Because much of the gas lies at radii where V(R) is nearly constant, most of the emission is crowded into two peaks near the extreme velocities. This double-horn profile is characteristic of galaxies where the rotation curve first rises, then remains roughly flat; the separation of the peaks is $W \approx 2V_max \sin i$.

VIII. INTERSTELLAR MEDIUM

Table 2.4 A 'zeroth-order' summary of the Milky Way's interstellar medium (after J. Lequeux)

Component	*Description*	*Density* (cm^{-3})	*Temperature* (K)	*Pressure* (p/k_B)	*Vertical extent*	*Mass* ($\mathcal{M}_\odot$)	*Filling factor*
Dust grains						10^7–10^8	Tiny
large $\lesssim 1\ \mu m$	Silicates, soot		~20		150 pc		
small ~ 100 Å	Graphitic C		30–100				
PAH < 100 atoms	Big molecules				80 pc		
Cold clumpy gas	Molecular: H_2	> 200	< 100	Big	80 pc	$(2) \times 10^9$	<0.1%
	Atomic: HI	25	50–100	2 500	100 pc	3×10^9	2%–3%
Warm diffuse gas	Atomic: HI	0.3	8 000	2 500	250 pc	2×10^9	35%
	Ionized: HII	0.15	8 000	2 500	1 kpc	10^9	20%
HII regions	Ionized: HII	1–10^4	~10 000	Big	80 pc	5×10^7	Tiny
Hot diffuse gas	Ionized: HII	~0.002	$\sim 10^6$	2 500	~5 kpc	(10^8)	45%
Gas motions	$\frac{3}{2}\langle \rho_{HI} \rangle \sigma_r^2$	$\langle n_H \rangle \sim 0.5$	$10\ km\ s^{-1}$	8 000			
Cosmic rays	Relativistic	$1\ eV\ cm^{-3}$		8 000	~3 kpc	Tiny	
Magnetic field	$B \sim 5\ \mu G$	$1\ eV\ cm^{-3}$		8 000	~3 kpc		
Starlight	$\langle \nu h_P \rangle \sim 1\ eV$	$1\ eV\ cm^{-3}$			~500 pc		
UV starlight	11–13.6 eV	$0.01\ eV\ cm^{-3}$					

Note: ( ) denotes a very uncertain value. Pressures and filling factors refer to the disk midplane near the Sun; notice that the pressures from cosmic rays, in magnetic fields, and the turbulent motions of gas clouds are roughly equal.

FIG. 1: interstellar medium

IX. VIRIAL THEOREM

$$\frac{1}{2}\frac{\mathrm{d}^2 I_{jk}}{\mathrm{d}t^2} = 2T_{jk} + \Pi_{jk} + W_{jk} + \Sigma_{jk}$$

This is the tensor virial theorem. Taking the trace on both sides of this equation gives

$$\frac{1}{2}\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} = 2K + W + \Sigma$$

where

$$I = \mathrm{Tr}\left(I_{ij}\right) = \int \rho r^2 \, \mathrm{d}^3\mathbf{x}, K = \mathrm{Tr}\left(K_{ij}\right) = \frac{1}{2}\int \rho \left\langle v^2 \right\rangle \mathrm{d}^3\mathbf{x}$$
$$W = \mathrm{Tr}\left(W_{ij}\right) = -\int \rho \mathbf{x}\cdot\nabla\Phi \mathrm{d}^3\mathbf{x}, \Sigma = \mathrm{Tr}\left(\Sigma_{ij}\right) = -\int \rho \left\langle v^2 \right\rangle \mathbf{x}\cdot \mathrm{d}\mathbf{S}$$

Note that I is the moment of inertia, K is the kinetic energy of the system, and Σ is the work done by the external pressure. The potential energy, W, is equal to the gravitational energy of the system only if any mass outside the surface S can be ignored in the computation of the potential.it is the important scalar virial theorem.

For a static system,$\frac{1}{2}\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} = 0$,and

$$2K + W + \Sigma = 0$$

giving a constraint on the global properties of any system in a static state. Finally, for $\Sigma = 0$ we have

$$E = -K = \frac{W}{2}$$

where $E = K + W$ is the total energy.

X. DYNAMICAL FRICTION

- Dynamical friction time: This is the time scale on which a satellite object in a large halo loses its orbital energy and spirals to the center. This time scale is proportional to M_{sat}/M_{main}, where M_{sat} is the mass of the satellite object and M_{main} is that of the main halo. Thus, more massive galaxies will merge with the central galaxy in a halo more quickly than smaller ones.

Dynamical friction removes energy from the forward motion of two passing galaxies, transferring it to the random motions of their stars. Long afterward, though, the combination of rotational motion and random speeds within each galaxy will be lower than before the collision. To see why, suppose that, before the encounter, the internal kinetic energy of one of the galaxies was KE_0. By the virial theorem, the potential energy $PE_0 = 2KE_0$; so its internal energy E_0 must be

$$E_0 = KE_0 + PE_0 = -KE_0$$

Dynamical friction increases the energy in random stellar motions, and hence the galaxy's internal energy, by δKE. The system is less strongly bound, so it expands. Long after the encounter, when it is once again in virial equilibrium, the kinetic energy is less than before.

XI. SURFACE BRIGHTNESS PROFILE OF SPHEROIDAL GALAXIES

The surface brightness profile of **spheroidal galaxies** is generally well fit by the Sersic profile, or $R^{1/n}$ profile,

$$I(R) = I_0 \exp\left[-\beta_n \left(\frac{R}{R_\mathrm{e}}\right)^{1/n}\right] = I_\mathrm{e} \exp\left[-\beta_n \left\{\left(\frac{R}{R_\mathrm{e}}\right)^{1/n} - 1\right\}\right]$$

where $I_0$is the central surface brightness,n is the so-called Sersic index which sets the concentration of the profile, R_e is the effective radius that encloses half of the total light, and $I_e = I(R_e)$. Surface brightness profiles are often expressed in terms of $\mu \sim 2.5 log(I)$(which has the units of mag $arcsec^2$), for which the Sersic profile takes the form

$$\mu(R) = \mu_{\mathrm{e}} + 1.086\beta_n \left[\left(\frac{R}{R_{\mathrm{e}}} \right)^{1/n} - 1 \right]$$

The value for β_n follows from the definition of R_e and is well approximated by $\beta_n = 2n - 0.324$ (but only for $n \geq 1$).

XII. LUMINOSITY FUNCTIONS AND MASS FUNCTIONS

A. Luminosity functions

The luminosity function $\Phi(M_V)$ describes how many stars of each luminosity are present in each pc^3: $\Phi(M_V)\Delta M_V$ is the density of stars with absolute V-magnitude between M_V and $M_V + \Delta M_V$.

$$\Phi(x) = \frac{\text{number of stars with } M_V - 1/2 < x < M_V + 1/2}{\text{volume } \mathcal{V}_{\max}\left(M_V\right) \text{ over which these could be seen}}$$

total luminosity of a galaxy depends strongly on whether it has recently been active in making these massive short-lived stars.

B. mass–luminosity relation

All main-sequence stars are burning hydrogen into helium in their cores. For any particular spectral type, these stars have nearly the same mass and luminosity, because they have nearly identical structures: the hottest stars are the most massive, the most luminous, and the largest. Main-sequence stars have radii between $0.1R_\odot$ and about $25R_\odot$: very roughly,

$$R \sim R_\odot \left(\frac{\mathcal{M}}{\mathcal{M}_\odot} \right)^{0.7} \quad \text{and} \quad L \sim L_\odot \left(\frac{\mathcal{M}}{\mathcal{M}_\odot} \right)^{\alpha}$$

where $\alpha \approx 5$ for $\mathcal{M} \lesssim \mathcal{M}_\odot$, and $\alpha \approx 3.9$ for $\mathcal{M}_\odot \lesssim \mathcal{M} \lesssim 10\mathcal{M}_\odot$. For the most massive stars with $\mathcal{M} \gtrsim 10\mathcal{M}., L \sim 50L_\odot \left(\mathcal{M}/\mathcal{M}_\odot\right)^{2.2}$.

Using models of stellar evolution, we can work backward from the present-day stellar population to find how many stars were born with each mass. We can calculate the initial luminosity function if we assume that the disk has been forming stars at a uniform rate throughout its history.

C. initial mass function

We can convert the initial luminosity function $\Psi(\mathcal{M})$ into an **initial mass function**: $\xi(\mathcal{M})\Delta\mathcal{M}$ is the number of stars that have been born with masses between $\mathcal{M}$ and $\mathcal{M} + \Delta\mathcal{M}$. Near the Sun, a good approximation for stars more massive than $\sim 0.5\mathcal{M}_\odot$ is

$$\xi(\mathcal{M})\Delta\mathcal{M} = \xi_0 \left(\mathcal{M}/\mathcal{M}_\odot\right)^{-2.35} \left(\Delta\mathcal{M}/\mathcal{M}_\odot\right)$$

where the constant ξ_0 sets the local stellar density; this is called the Salpeter initial mass function. . If we understood better how stars form, we might be able to predict the initial mass function.

XIII. BALMER JUMP

The Balmer lines of hydrogen are relatively weak because hydrogen is almost totally ionized. Note that the flux decreases sharply at wavelengths less than $3800A$, this is called the Balmer jump.

XIV. GUNN–PETERSON EFFECT

For high redshift quasar,such as $1425 + 6039$ with $z = 3.173$:broad $Ly\alpha$ emission at $1216A$ is redshifted to the visible region. At shorter wavelengths, narrow absorption lines of the $Ly\alpha$ forest are dense. we see that forest clouds can remove a substantial fraction of the quasar's light just shortward of its $Ly\alpha$ emission line, leaving the average intensity lower than that on the long-wavelength side of the emission line: the Gunn–Peterson effect. all the light just shortward of the $Ly\alpha$ emission line is missing, having been absorbed by diffuse neutral gas at $z \geq 5.8$.

XV. THE FABER–JACKSON RELATION

The range in the velocity dispersion σ for elliptical galaxies is close to what we saw for the peak rotation speed of disk galaxies. Just as for spirals, the stars move faster in more luminous galaxies. At the centers of bright ellipticals, the dispersion can reach $500\mathrm{kms}^{-1}$, while in the least luminous objects $\sigma \approx 50\mathrm{kms}^{-1}$. Roughly $L \propto \sigma^4$, this is often called the **Faber–Jackson relation**. In the V band, roughly

$$\frac{L_V}{2 \times 10^{10} L_\odot} \approx \left(\frac{\sigma}{200 \text{ km s}^{-1}}\right)^4 .$$

Like the Tully–Fisher relation for spirals, the Faber–Jackson relation can be used to estimate a galaxy's distance from its measured velocity dispersion.

XVI. THE FUNDAMENTAL PLANE

Another possibility is to use the fundamental plane relation. The elliptical galaxies of the Coma cluster all lie close to a plane in the three-dimensional 'space' of central velocity dispersion σ, effective radius R_e, and surface brightness $I_e = I(R_e)$. Approximately, we have

$$R_\mathrm{e} \propto \sigma^{1.2} I_\mathrm{e}^{-0.8}$$

the fundamental plane relation reflects some basic processes, still to be understood, by which elliptical galaxies form.

XVII. AGE–METALLICITY DEGENERACY

The main goal of the spectral evolution models described above is to use the observed spectra of galaxies to constrain their age and metallicity, or, more precisely, their star-formation history and element abundances. Unfortunately, there is a fundamental limitation to this, which **arises from a degeneracy between stellar age and metallicity**, the evolution of a star with a given initial mass in the $L - T_eff$ diagram depends on its chemical composition: stars with higher metallicities evolve faster. As a result, a population of young stars with a relatively high metallicity has a SED that looks very similar to that of an older population with lower metallicity. In order to break this age–metallicity degeneracy one has to use additional information that probes the fine structure of the SED. The spectral indices defined above are ideally suited for this. The strength of the Balmer lines, such as $H\beta$ and $H\gamma$, are more sensitive to stellar age, while the metal line strengths, such as Mgb, Fe_1 and Fe_2, are more sensitive to metallicity. Therefore, measurements of these line indices can be used to break the age–metallicity degeneracy.

XVIII. BIMODALITY

The bimodality of the galaxy population is also evident from the color–magnitude relation, the galaxy population is divided into a red sequence and a blue sequence (also sometimes called the blue cloud). Two trends are noteworthy. First of all, at the bright end the red sequence dominates, while at the faint end the majority of the galaxies are blue. . As we will see this most likely reflects that the **stellar populations in brighter galaxies are both older and more metal rich**, although it is still unclear which of these two effects dominates, and to what extent dust plays a role.

XIX. MEASUREMENTS OF MASS