ELEYANG DESIGN

La Vita E Bella

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Chapter Ⅰ

Title 质点运动学

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§. 时间与空间

Ⅰ. 自然界中的时间量级

- ▲人体心律周期 0.8 s
- ▲太阳光传播到地球 5×10^2 s
- ▲地球年龄 46亿年　太阳年龄 50亿年　宇宙年龄 150亿年
- ▲人眼视觉暂留时间 0.1 s　人体感觉神经脉冲间隔 1 ms

Ⅱ. 自然界中的长度量级

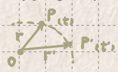
- ▲ $R_{⊕} = 6371$ km　$D_{moon} = 3477$ km　$D_{⊙} = 1.4\times10^6$ km
- ▲ 日地距离 1.5×10^8 km
- ▲ 1光年 9.5×10^{12} km
- ▲ 可见光波长 0.4 ~ 0.7 μm
- ▲ 原子半径 0.1 nm　原子核半径 1 fm (10^{-15} m)

Ⅲ. 天文单位

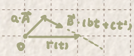
天文学中 日地平均距离 1.5×10^8 km 为一个天文单位

§. 位置矢量与轨道方程

▶ **[位置矢量]** 由参考点 O 指向质点瞬时位置 P 所确定的矢量　$\vec{r}(t)$



[Ex] $\vec{r}(t) = a\vec{A} + (bt + ct^2)\vec{B}$　求轨道形态特征



轨道为一条直线

方向沿 $\vec{B}$　距离 $d = aA\sin\theta$

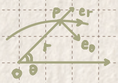
[轨道方程] $\vec{r} = \vec{r}(t)$　//　$\begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases}$

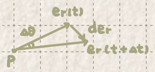
▶ **[速度矢量]** 位移矢量 $\Delta\vec{r} = \vec{r}(t+\Delta t) - \vec{r}(t)$

§. 极坐标系

▶ **[极坐标系运动方程]** 已知质点运动轨迹方程 $r = r(t)$　$\theta = \theta(t)$　$\vec{e}_r$　$\vec{e}_\theta$ 随 t 变化

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d(r\vec{e}_r)}{dt} = \frac{dr}{dt}\vec{e}_r + r\frac{d\vec{e}_r}{dt}$$



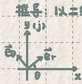


$$d\vec{e}_r = d\theta\cdot\vec{e}_\theta$$

$$\Rightarrow \frac{d\vec{e}_r}{dt} = w\vec{e}_\theta \qquad \frac{d\vec{e}_\theta}{dt} = -w\vec{e}_r$$

then $\vec{v} = \underbrace{\frac{dr}{dt}\vec{e}_r}_{\vec{v}_r} + \underbrace{r\frac{d\theta}{dt}\vec{e}_\theta}_{\vec{v}_\theta}$

推导：以二维为例



$$\vec{e}_r = \cos\theta\,\vec{i} + \sin\theta\,\vec{j}$$

$$\vec{e}_\theta = -\sin\theta\,\vec{i} + \cos\theta\,\vec{j}$$

$$\frac{d\vec{e}_r}{dt} = (-\sin\theta\,\vec{i} + \cos\theta\,\vec{j})\,\dot{\theta}$$

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Chapter V

Title 角动量变化定理与角动量守恒 Date /

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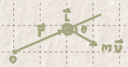
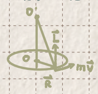
§. 角动量

▶【角动量】 $\vec{L} = \vec{r} \times m\vec{v}$ 大小 $rmv\sin\theta$ 方向：右手螺旋



角动量守恒与参考点有关



此时O下 $\vec{L} = mR^2 \cdot \vec{w}$ O'下 L不守恒

$$(L_x, L_y, L_z) = \begin{vmatrix} i & j & k \\ x & y & z \\ P_x & P_y & P_z \end{vmatrix}$$

▶【力矩】 $\vec{M} = \vec{r} \times \vec{F}$ 大小：$rF\sin\alpha$ 方向：右手螺旋

$$(M_x, M_y, M_z) = \begin{vmatrix} i & j & k \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

§. 角动量变化定理

#推导：

$$\frac{d\vec{L}}{dt} = \frac{d(\vec{r} \times m\vec{v})}{dt} = \left(\frac{\vec{r} \times dm\vec{v}}{dt} + \frac{d\vec{r} \times m\vec{v}}{dt}\right)$$

$$= \vec{r} \times \vec{F} + \vec{v} \times m\vec{v} = \vec{r} \times \vec{F} = \vec{M}$$

$\Rightarrow \vec{L_2} - \vec{L_1} = \int_{t_1}^{t_2} \vec{M} dt$ 角动量增量 = 力矩的积分

$\Delta L = M\Delta t$ 类比 $\Delta p = f\Delta t$

力矩 = 0（F 朝向参考点、或不受力） 角动量守恒 // i方向力矩分量 = 0 角动量分量守恒

§. 有心运动

【有心运动】满足运动物体机械能守恒 角动量守恒（对力心守恒）

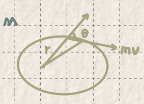
有心力场 $F \propto r^n e_r$

角动量守恒：方向：轨道必须限定在一平面内 且包含力心 M

数值：掠面速度 $v_s = \frac{\Delta S}{\Delta t}$ 不变

$dS = \frac{1}{2} r \cdot vdt \cdot \sin\theta$

$v_s = \frac{1}{2} rv\sin\theta = \frac{|\vec{L}|}{2m}$



▶ 太阳恒星

平方反比有心力场：$E<0$ 椭圆 $E=0$ 抛物线 $E>0$ 双曲线

1° $\frac{1}{2}mv_1^2 - G\frac{Mm}{r_1} = \frac{1}{2}mv_2^2 - G\frac{Mm}{r_2} \Rightarrow v_2^2 - v_1^2 = 2GM\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

2° $r_1 m v_1 = r_2 m v_2 \Rightarrow \frac{v_1}{v_2} = \frac{r_2}{r_1}$

Summary 总结

Chapter VI

Title 质心力学定理

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§. 质心动量定理

▶【质心】质点组质心位置矢量 $\vec{r}_c = \frac{\Sigma(m_i \vec{r}_i)}{M}$

(质点质量加权后平均)

【动量】质心动量 = 质点组总动量 质心动量的改变量等于合外力的冲量 简单加和

$$\vec{v}_c = \frac{d\vec{r}_c}{dt} = \frac{d}{dt}\left(\frac{\Sigma m_i \vec{r}_i}{M}\right) = \frac{1}{M}\Sigma(m_i v_i)$$

▶【质心参考系】以质心为参考点

§. 质心动能定理

【动能】质点组总动能 = 质心动能 + 相对质心动能 科尼西定理 总动能 = 质心动能 $(\frac{1}{2}Mv_c^2)$ + 各质点相对质心动能 $(\frac{1}{2}m_i v_{ic}^2)$

建立质心系：$\vec{r}_i = \vec{r}_c + \vec{r}_{ic}$ $\vec{v}_i = \vec{v}_c + \vec{v}_{ic}$

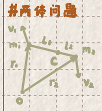
$$E_k = \Sigma \frac{1}{2} m_i v_i^2 = \Sigma \frac{1}{2} m_i (\vec{v}_c + \vec{v}_{ic}) \cdot (\vec{v}_c + \vec{v}_{ic})$$

$$(\vec{v}_c + \vec{v}_{ic}) \cdot (\vec{v}_c + \vec{v}_{ic}) = \vec{v}_c^2 + \vec{v}_{ic}^2 + 2\vec{v}_c \cdot \vec{v}_{ic}$$

$$= \Sigma \frac{1}{2} m_i \vec{v}_c^2 + \Sigma \frac{1}{2} m_i \vec{v}_{ic}^2 + \underbrace{\Sigma m_i \vec{v}_c \cdot \vec{v}_{ic}}_{\text{质心系中总动量为0}}$$

$$= \frac{1}{2} M v_c^2 + \Sigma \frac{1}{2} m_i \vec{v}_{ic}^2 = E_c + E_{rc}$$

#两体问题



已知 $m_1 l_1 = m_2 l_2$

m_1 相对 m_2 位矢 $\vec{l} = \vec{l}_1 - \vec{l}_2$

$\vec{v}_{1c} = \frac{d\vec{l}_1}{dt}$ $\vec{v}_{2c} = \frac{d\vec{l}_2}{dt}$

$$E_{rc} = \frac{1}{2} m_1 \left|\frac{dl_1}{dt}\right|^2 + \frac{1}{2} m_2 \left|\frac{dl_2}{dt}\right|^2$$

$$= \frac{1}{2} m_1 \left(\frac{m_2}{m_1+m_2} \frac{dl}{dt}\right)^2 + \frac{1}{2} m_2 \left(\frac{m_1}{m_1+m_2} \frac{dl}{dt}\right)^2$$

$$= \frac{1}{2} \frac{m_1 m_2}{m_1+m_2} \left(\frac{dl}{dt}\right)^2 = \frac{1}{2}\left(\frac{m_1 m_2}{m_1+m_2}\right)(\vec{v}_1 - \vec{v}_2)^2$$

$= \frac{1}{2}\mu v_r^2$ (μ 为约化质量；v_r 为相对速度) (两体碰撞：$\Delta E_k = -\frac{1}{2}(1-e^2)\mu v_r^2$)

§. 质心角动量定理

【角动量】质点组总角动量 = 质心角动量 + 相对质心角动量

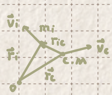
建立质心系：$\vec{r}_i = \vec{r}_c + \vec{r}_{ic}$ $\vec{v}_i = \vec{v}_c + \vec{v}_{ic}$

$$\vec{L} = \Sigma(\vec{r}_i \times m\vec{v}_i) = \Sigma(\vec{r}_c + \vec{r}_{ic}) \times (\vec{v}_c + \vec{v}_{ic}) \cdot m_i$$

$$= \vec{r}_c \times \vec{v}_c + \vec{r}_{ic} \times \vec{v}_{ic} + \vec{r}_c \times \vec{v}_{ic} + \vec{r}_{ic} \times \vec{v}_c$$

$$M \cdot \vec{r}_c \times \vec{v}_c \qquad \Sigma m_i \vec{r}_{ic} \times \vec{v}_{ic} \qquad \vec{r}_c \times \Sigma m_i \cdot \vec{v}_{ic} = 0 \qquad -\vec{v}_c \times \Sigma m_i \vec{r}_{ic} = 0$$

$$= L_c + L_{rc}$$



▶【质心角动量变化定理】$\frac{dL_c}{dt} = \vec{M}_c = \vec{r}_c \times \Sigma F_i$

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Chapter VII

Title 刚体力学(上)　　**Date** /

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§. 刚体运动学

▶【刚体】无形变(劲度系数极大)

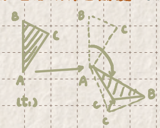
① ∀两点PQ V_P V_Q 在PQ方向投影分量总是相等

② 刚体内部一对内力做功之和恒为0

刚体 ∀不共线三个点 可以确定整体刚体空间方位 (刚体三角形)

【平动】各点 $\vec{v}(t)$ $\vec{a}(t)$ 相等　$M\vec{a}_c = \sum F_i$

自由刚体自由度 (描述对象运动位置的独立坐标)



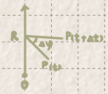
① A：三个平动自由度

② B：二个转动自由度

③ C：一个转动自由度

【转动】定点转动　定轴转动

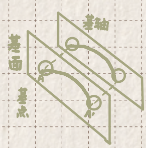
①定轴转动



$\vec{v} = \vec{w} \times \vec{r}$

$\vec{a} = \dot{\vec{w}} \times \vec{r} + \vec{w} \times \vec{v}$　(刚体只有1个 $\vec{w}$)

②平面运动　∀点运动轨道限定于一定平面内 且彼此平行或重合



① 选定基面　化三维为二维

② 选定基点　分解：基点运动 + 绕基点转动 (2+1)

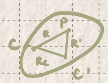
③ 回归基轴　分解：基轴平动 + 绕基轴转动

一般选基轴通过质心

=> 【V】基点为C，$\vec{v}_c$　则该基面上各点 $\vec{v} = \vec{v}_c + \vec{w} \times \vec{R}$

$\vec{v}_A = \vec{v}_c + \vec{w} \times \vec{R}_A$

$\vec{v}_B = \vec{v}_c + \vec{w} \times \vec{R}_B$



【W】w与基点选择无关

prof: $\vec{v}_P = \vec{v}_c + \vec{w} \times \vec{R}$

$= \vec{v}_c' + \vec{w}' \times \vec{R}'$

$\vec{v}_c' = \vec{v}_c + \vec{w} \times \vec{R}_c$

$\vec{R} = \vec{R}_c + \vec{R}'$

$\Rightarrow \vec{v}_c + \vec{w} \times \vec{R} = \vec{v}_c + \vec{w} \times (\vec{R} - \vec{R}') + \vec{w}' \times \vec{R}'$

$\Rightarrow \vec{w}' \times \vec{R}' = \vec{w} \times \vec{R}'$

$\vec{w}' = \vec{w}$

【瞬心】基面上必∃P s.t. $\vec{v}_p = 0$　// $\vec{v}_c + \vec{w} \times \vec{R}_0 = 0$

刚体在一个时刻只有一个(属于整个刚体的)角速度。

§. 刚体平衡条件

▶【刚体平衡】$\sum \vec{F}_{ext} = 0$　(平动平衡)

$\sum \vec{M} = 0$　(转动平衡)

【重心】以力心为原点 总力矩为0

$\sum_i r_i \times (m_i g_i) = \vec{r}_{cog} \times \sum_i m_i g_i$

【稳定性问题】势能曲线 $U(\theta)$

$U' = 0$ 时平衡 ⇒ $U'' > 0$　稳定

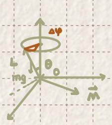
$U'' = 0$　随遇

$U'' < 0$　不稳定

Summary 总结

关于角动量进动问题

【力矩 ⊥ 角动量】当 ΔL 很小 或 $M \perp L$ 时：转动轴本身在转动



如图 重力矩 $\vec{M} = \vec{r} \times m\vec{g}$　(非定轴转动)

$|M| = rmg\sin\theta$

$\Delta L = L\sin\theta\,\Delta\varphi$

$= I w \sin\theta\,\Delta\varphi$

$\Omega = \lim_{\Delta t \to 0} \frac{\Delta\varphi}{\Delta t} = \frac{mgr}{Iw}$　// 条件：$w \gg \Omega$ 或 $Iw \gg mgr$

Content

Title 刚体力学(中)

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§. 转动惯量

▸【角动量的轴分量】 $\vec{L}_{总}=\sum(\vec{r}_i\times dm_i\vec{v}_i)=\sum dm\,\vec{r}_i\times(\vec{w}\times\vec{r}_i)$ (定轴转动)

其中 $r_i=(x_i\ y_i\ z_i)$ $\vec{w}=(0,0,w)$

$\vec{L}=\sum dm\,(-x_iz_iw,\ -y_iz_iw,\ (x_i^2+y_i^2)w)$

$$\Rightarrow\begin{cases}L_x=w\cdot\sum(-x_iz_i\Delta m_i)\\ L_y=w\cdot\sum(-y_iz_i\Delta m_i)\\ L_z=w\cdot\sum(x_i^2+y_i^2)\Delta m_i\end{cases}$$

▸【转动惯量】 $I=\sum(x_i^2+y_i^2)\Delta m_i=\sum R_i^2\Delta m_i$

其中 R_i 为轴距

L_z(轴分量) $=Iw$ 此处类比 $p=mv$ m 惯性质量

$$I=\begin{cases}\sum R_i^2\Delta m_i & 质点组\\ \int R^2\eta\,dl & 质量线分布\\ \iint R^2\sigma\,dS & 面分布\\ \iiint R^2\rho\,dV & 体分布\end{cases}$$

Ex: 

$I=\int_{-\frac{L}{2}}^{\frac{L}{2}}x^2\eta\,dx=\frac{m}{L}\cdot2\cdot\frac{1}{3}(\frac{L}{2})^3=\frac{1}{12}mL^2$

$I=mR^2$ $I=\frac{1}{2}mR^2$ $I=\frac{2}{3}mR^2$ $I=\frac{2}{5}mR^2$ $I=\frac{1}{4}mR^2+\frac{1}{12}ml^2$

* 平行轴定理 与质心轴平行的转轴: $I=I_c+md^2$
* 薄板正交轴定理 质量呈面分布 薄板 xy 绕 x 轴 I_x 绕 y 轴 I_y : $I_z=I_x+I_y$

Ex:  $I_x=I_y=\frac{1}{2}I_z$

$I_z=mR^2$

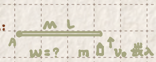
$I=\frac{1}{3}m_2d^2+\frac{1}{2}m_1R^2+m_1(R+d)^2$

§. 定轴转动定理

▸【Theorem1】 $I\cdot\dot{\vec{w}}=M_z$

$\frac{dL_z}{dt}=M_z$ $L_z=Iw\Rightarrow\frac{d(Iw)}{dt}=M_z$

【角动量 L_z 守恒】 $M_z=0$ 时 $L_z=Iw$ 守恒

Ex: 

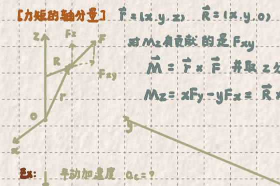
动能不守恒(f) 动量不守恒($f_{轴}$)

$LmV_0=Iw$

求I 

$I=\frac{1}{3}ML^2+mL^2$ $w=\frac{mV_0}{(m+\frac{1}{3}M)L}$

【力矩的轴分量】 $\vec{r}=(x,y,z)$ $\vec{R}=(x,y,0)$



对 M_z 有贡献的是 F_{xy}

$\vec{M}=\vec{r}\times\vec{F}$ 并取 z 分量

$M_z=xF_y-yF_x=\vec{R}\times\vec{F}_{xy}$

Ex: 平动加速度 $a_c=?$

平动+转动 $V_c=wR$ $(V_{bc}=wR)$

故 $V_b=0$ (纯滚模型)

$$\begin{cases}Mg-T=a_cM & 质心定理\\ I\cdot\dot{\vec{w}}=T\cdot R & Th.1\ 定轴转动定理\end{cases}$$

$a_c=\dot{\vec{w}}\cdot R$ $\Rightarrow a_c=\frac{mR^2g}{I+mR^2}=\frac{2}{3}g$

▸【Theorem2】 $E_k=\frac{1}{2}\sum\Delta m_iv_i^2=\frac{1}{2}(\sum\Delta m_iR_i^2)w^2=\frac{1}{2}Iw^2$

▸【Theorem3】 $dE_k=d(\frac{1}{2}Iw^2)=wIdw$ $Idw=M_zdt$ $\Rightarrow dE_k=M_zwdt$

$\Rightarrow\frac{1}{2}Iw_2^2-\frac{1}{2}Iw_1^2=\int_1^2M_zd\varphi$ (其中 $d\varphi$ 为力的作用点在 dt 中角位移)

Summary 总结

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Title 刚体力学习题（下）　　Date /

Keywords 关键词	Notes 笔记	Review 复习记录 / / / / /

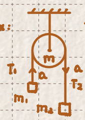
二人抬杠一方撒手

Ex: 

$N = ?$

$$\begin{cases} I\dot{w} = mg\frac{l}{2} \\ ma_c = mg - N \\ a_c = \dot{w}\cdot\frac{l}{2} \end{cases} \Rightarrow N = \frac{1}{4}mg$$

滑轮运动

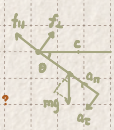
Ex: 

T_1　$T_2 = ?$

$$\begin{cases} T_1 - m_1 g = am_1 \quad m_2 g - T_2 = am_2 \\ I\cdot\dot{w} = (T_2 - T_1)R \\ a = \dot{w}\cdot R \qquad I = \frac{1}{2}mR^2 \end{cases} \Rightarrow \begin{cases} a = \dfrac{m_2 - m_1}{m_2 + m_1 + \frac{m}{2}} \\ T_1 = \dfrac{m_1(2m_2 + \frac{m}{2})}{m_1 + m_2 + \frac{m}{2}}g \quad T_2 = \dfrac{m_2(2m_1 + \frac{m}{2})}{m_1 + m_2 + \frac{m}{2}}g \end{cases}$$

铅直面内转动杆

Ex: 



$w(\theta) = ?$　$\frac{dw}{dt}(\theta) = ?$

$a_c(\theta) = ?$　a_{cn}　$a_{c\tau} = ?$

$f(\theta) = ?$

①受力：f　mg

②动量 x　角动量 x　机械能 √

(a) $mg\frac{l}{2}\sin\theta = \frac{1}{2}Iw^2(\theta)$　　$I = \frac{1}{3}mR^2$

$\Rightarrow w(\theta) = \sqrt{\frac{3g\sin\theta}{L}}$

(b) $a_{cn} = \frac{l}{2}w^2 = \frac{3}{2}g\sin\theta$

$a_{c\tau} = \frac{l}{2}\frac{dw}{dt} = \frac{3}{4}g\cos\theta$

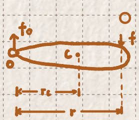
$$\frac{dw}{dt} = \frac{dw}{d\theta}\frac{d\theta}{dt} = w\cdot\sqrt{\frac{3g}{L}}\frac{\cos\theta}{2\sqrt{\sin\theta}} = \frac{3g\cos\theta}{2L}$$

(c) $f_\perp$: $mg\cos\theta - f_\perp = ma_{c\tau}$

$f_{//}$: $f_{//} - mg\sin\theta = ma_{cn}$

$$\Rightarrow \begin{cases} f_{//}(\theta) = \frac{5}{2}mg\sin\theta \\ f_\perp(\theta) = \frac{1}{4}mg\cos\theta \end{cases}$$

打击中心

Ex: 

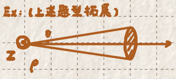
$$\begin{cases} f\cdot r = \frac{dw}{dt}\cdot I \\ f - f_0 = a_c m \\ a_c = \frac{dw}{dt}\cdot r_c \end{cases} \Rightarrow f_0 = \left(1 - \frac{mr_c r}{I}\right)f$$

r满足何种条件 f_0最小？

$r_0 = \frac{I}{mr_c} = kr_c$　（其中 $k = \frac{I}{mr_c^2}$）

圆锥棒

Ex:（上述题型拓展）



$r_c = ?$　$I = ?$

$x \to x + \Delta x$: $\Delta V = \pi r^2\cdot\Delta x = \pi(x\tan\theta)^2\Delta x$

$\Delta m = \rho\Delta V = \pi\rho\tan^2\theta\, x^2\Delta x$

$\eta(x) = \pi\rho\tan^2\theta\, x^2$

① $r_c = \dfrac{\int_0^L \eta(x)x\,dx}{\int_0^L \eta(x)dx} = \dfrac{\frac{1}{4}AL^4}{\frac{1}{3}AL^3} = \frac{3}{4}L$

② $dI_1 = \frac{1}{2}\cdot\frac{1}{2}mR^2 = \frac{1}{4}\eta(\tan\theta x)^2dx$　Φ

$dI_2 = mx^2 = \eta x^2dx$

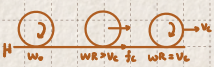
$I = \int_0^L dI = \int_0^L \left(1 + \frac{1}{4}tg^2\theta\right)\eta x^2dx$

$= \frac{3}{5}mL^2\left(1 + \frac{1}{4}tg^2\theta\right)$

Keywords 关键词	Notes 笔记

从纯转到纯滚

Ex: m R 求 $t=?$ $l=?$ $w_p=?$ $\Delta E_k=?$



$$\begin{cases} v_c=\mu g t \\ w=w_0-\frac{dw}{dt}t \\ I\frac{dw}{dt}=\mu m g R \end{cases} \Rightarrow \begin{cases} t=\frac{w_0R}{\mu g(1+\frac{mR^2}{I})} \\ l=\frac{w_0^2R^2}{2\mu g(1+\frac{mR^2}{I})^2} \\ v_c=\frac{w_0R}{1+\frac{mR^2}{I}} \end{cases}$$

(纯滚条件 $v_c=wR$)

$w_p=\frac{w_0}{1+\frac{mR^2}{I}}$

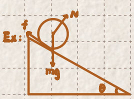
圆盘/圆柱 $I=\frac{1}{2}mR^2$

圆环/圆筒 $I=mR^2$

从纯滑到纯滚同理

斜面上圆筒滑动

Ex:



下落 h v_c w $f=?$

$$\begin{cases} mgh=\frac{1}{2}mv_c^2+\frac{1}{2}Iw^2 \\ v_c=wR \end{cases}$$

$$\Rightarrow v_c=\sqrt{\frac{2gh}{1+\frac{I}{mR^2}}}=\begin{cases} \frac{2}{\sqrt{3}}\sqrt{gh} & 实心 \\ \sqrt{gh} & 空心 \end{cases} \qquad w=\begin{cases} \frac{2}{\sqrt{3}R}\sqrt{gh} & 实心 \\ \frac{\sqrt{gh}}{R} & 空心 \end{cases}$$

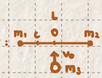
$$\begin{cases} mg\sin\theta-f=a_c \\ v_c^2=2a_c\frac{h}{\sin\theta} \end{cases} \Rightarrow f=\begin{cases} \frac{1}{3}mg\sin\theta & 实 \\ \frac{1}{2}mg\sin\theta & 空 \end{cases}$$

满足纯滚条件 $\mu\geq\frac{1}{3}tg\theta$ // $\mu\geq\frac{1}{2}tg\theta$

法2
$$\begin{cases} I\frac{dw}{dt}=fR \\ ma_c=mg\sin\theta-f \\ v_c=Rw \Rightarrow a_c=R\frac{dw}{dt} \end{cases}$$

水平面内球杆碰撞

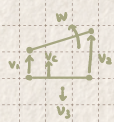
Ex:



碰后 $v_c=?$

$w=?$

反弹: $v_3=?$



① 动能守恒: $\frac{1}{2}(m_1+m_2)v_c^2+\frac{1}{2}Iw^2+\frac{1}{2}m_3v_3^2=\frac{1}{2}m_3v_0^2$

② 动量守恒: $(m_1+m_2)v_c-m_3v_3=m_3v_0$

③ 角动量守恒: (以C为参考点) $I_cw-r_0m_3v_3=r_0m_3v_0$

(以O为参考点) $I_cw=(m_1+m_2)v_cr_0$

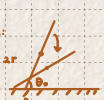
$r_0=\frac{m_1-m_2}{2(m_1+m_2)}l$ $\qquad I_c=\frac{m_1m_2}{m_1+m_2}l^2$

$$\Rightarrow \begin{cases} w=\frac{4(m_1-m_2)m_3}{4m_1m_2+(m_1+m_2)m_3}\frac{v_0}{l} \\ v_c=\frac{8m_1m_2m_3}{(m_1+m_2)[4m_1m_2+(m_1+m_2)m_3]}v_0 \end{cases}$$

光滑面上细杆倾倒

Ex:

m 2r



$v_c(\theta)=?$

$a_c(\theta)=?$

$w(\theta)=?$

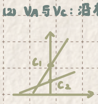
$v_A(\theta)=?$

$N(\theta)=?$

(1) 质心运动轨迹: 受力 mg ↓ N ↑ 合力 ↓

质心沿铅垂方向加速 ↓

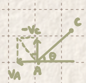
(2) v_A 与 v_c: 沿杆方向速度相同



$v_c\sin\theta=v_A\cos\theta$

$v_A=v_c\tan\theta$

(3) w 与 v_c



$\vec{v}_A=\vec{v}_c+w\times r$

$\Rightarrow w=\frac{v_c}{r\cos\theta}$

(4) 找出 v_c: $\vec{N}\cdot\vec{v}_A=P=0$ ⇒ 机械能守恒

$\frac{1}{2}mv_c^2+\frac{1}{2}Iw^2=mg(y_0-y)$ $\qquad w=\frac{v_c}{r\cos\theta}$ $\qquad I=\frac{m(2l)^2}{12}=\frac{ml^2}{3}$ $\qquad y_0-y=r(\sin\theta_0-\sin\theta)$

Keywords 关键词	Notes 笔记

$$\Rightarrow v_c = \sqrt{\frac{6gr(\sin\theta_0 - \sin\theta)\cos^2\theta}{1+3\cos^2\theta}}$$

(5) $a_c(\theta) = \frac{dv_c}{d\theta} \cdot (-\omega)$

$\omega = \frac{v_c}{r\cos\theta}$

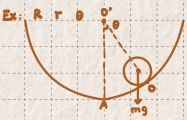
$$\Rightarrow a_c(\theta) = \frac{-v_c}{r\cos\theta}\frac{dv_c}{dt} = \frac{-1}{2r\cos\theta}\frac{dv_c^2}{d\theta}$$

$$= \frac{3(1+\cos^2\theta)\cos^2\theta + 2(\sin\theta_0\sin\theta - 1)}{(1+3\cos^2\theta)^2} 3g$$

(6) $N(\theta) = mg - a_c m = \frac{7 - 6\sin\theta_0\sin\theta - 3\cos^2\theta}{(1+3\cos^2\theta)^2} mg$

碗中小球

Ex: R r θ



以O为参考点

O为瞬心: N f 无力矩　　$mgr\sin\theta = I\frac{d\omega}{dt}$　　$v_c = \omega r \Rightarrow \frac{dv}{dt} = r\frac{d\omega}{dt}$

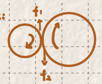
$v_c = (R-r)\frac{d\theta}{dt}$　　$\frac{dv_c}{dt} = (R-r)\frac{d^2\theta}{dt^2}$

则 $\frac{d^2\theta}{dt^2} = -\frac{mgr^2}{(R-r)I}\sin\theta \approx -\frac{mgr^2}{(R-r)I}\theta$ (简谐运动标准形式)　$\omega_0 = \sqrt{\frac{mgr^2}{(R-r)I}}$

$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{(R-r)I}{mgr^2}}$　　对O瞬时轴: $I = \frac{2}{5}mr^2 + mr^2 = \frac{7}{5}mr^2$

两轮磨合

Ex:



$(\omega_{10}\ I_1\ R_1)$ $(\omega_{20}\ I_2\ R_2)$

相接触后得 ω_1' ω_2' = ?

$I_1 d\omega_1 = f_1 R_1 dt$

$I_2 d\omega_2 = f_2 R_2 dt$　$\Rightarrow$　$I_1 R_2(\omega_1' - \omega_{10}) = I_2 R_1(\omega_2' - \omega_{20})$

$f_1 = f_2$　　$\omega_1' R_1 = -\omega_2' R_2$

$$\Rightarrow \omega_1' = \frac{R_2(I_1 R_2 \omega_{10} - I_2 R_1 \omega_{20})}{I_1 R_2^2 + I_2 R_1^2}$$

$$\omega_2' = \frac{R_1(I_2 R_1 \omega_{20} - I_1 R_2 \omega_{10})}{I_1 R_2^2 + I_2 R_1^2}$$

Summary 总结

【刚体力学三大公式】 ① $L_z = I\omega$

② $I\dot{\omega} = M_z$

③ $E_k = \frac{1}{2}I\omega^2$

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Title 振动(上) **Date** /

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§. 振动的描述

▶【简谐运动】一维弹簧振子有一初位移,释放后自由运动,出现周而复始的简谐运动

$x(t) = A\cos(wt+\varphi_0)$

$x(t) = A\cos(2\pi f t+\varphi_0)$

$x(t) = A\cos(\frac{2\pi}{T}t+\varphi_0)$

▶【傅里叶分析】一个周期为T的函数 $x(t)$ 可以被展开为一系列不同频率的简谐函数的叠加

$x(t) = x_0 + \sum_n C_n \cos(2\pi f_n t + \varphi_n)$ $n=1,2,3\cdots$

其中 $f_n = n f_1$ $f_1 = \frac{1}{T}$ (基频) C_n 为傅里叶系数 取决于 $x(t)$ 的线形

$$\begin{cases} x_0 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)dt \\ a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\cos(2\pi f_n t)dt \\ b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\sin(2\pi f_n t)dt \\ C_n = \sqrt{a_n^2+b_n^2} \quad \varphi_n = -\mathrm{arctg}\frac{b_n}{a_n} \end{cases}$$

§. 自由振动

弹簧振子

$f = -kx$ $m\ddot{x} = f$

$\Rightarrow \ddot{x} = -\frac{k}{m}x$ 令 $w_0^2 = \frac{k}{m}$ $w_0 = \sqrt{\frac{k}{m}}$ $\Rightarrow \ddot{x} = -w_0^2 x$

由常微分方程 $x(t) = A\cos(w_0 t + \varphi_0)$

▶【本征频率】即固有频率 w_0 $f_0 = \frac{w_0}{2\pi}$

【振幅、初相位】 $A = \sqrt{x_0^2 + \frac{v_0^2}{w_0^2}}$ $\varphi_0 = -\mathrm{arctg}\frac{v_0}{x_0 w_0}$ $\Rightarrow \begin{cases} x_0 = A\cos\varphi_0 \\ v_0 = -Aw_0\sin\varphi_0 \end{cases}$

复摆 单摆 等时摆

▶【复摆】一刚体可绕O轴在铅直面内作定轴转动(轴承摩擦不计)

刚体仅在重力矩作用下摆动

$I\frac{d^2\theta}{dt^2} = M_z$ $M_z = -r_c mg\sin\theta$ $(\sin\theta \approx \theta)$

$\Rightarrow \theta(t) = \theta_0\cos(w_0 t + \varphi_0)$ $w_0 = \sqrt{\frac{mgr_c}{I}}$

【单摆】考虑 $I = ml^2$ $r_c = l$ 忽略摆线质量 $\Rightarrow$ $w_0 = \sqrt{\frac{g}{l}}$ $T_0 = 2\pi\sqrt{\frac{l}{g}}$

▶【自由谐振子能量】

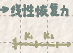
$$\left.\begin{aligned} E_k(t) &= \frac{1}{2}m(\frac{dx}{dt})^2 = \frac{1}{2}mA^2w_0^2\sin^2(w_0t+\varphi_0) \\ E_p(t) &= \frac{1}{2}kx^2 = \frac{1}{2}mA^2\cos^2(w_0t+\varphi_0) \end{aligned}\right\} \Rightarrow E(t) = \frac{1}{2}kA^2 = \frac{1}{2}mA^2w_0^2$$

等效劲度系数 求本征频率

【1】考察力 振子作一小位移 x

$f = -k_e x \xrightarrow{\checkmark}$ 线性恢复力 $(w_0 = \sqrt{\frac{k_e}{m}}$

$k_e = k_1 + k_2$



$k_e = \frac{k_1 k_2}{k_1 + k_2}$

【2】考察能量 $w_0 = \sqrt{\frac{2E}{mA^2}}$

Summary 总结

Title 振动(中) **Date** /

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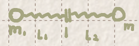
§. 多自由度弹性系统

▶ [双振子] 双原子分子振动模式



[法1] 系统不受外力 → 质心不动 → 双振子同频逆向运动 $A_1/A_2 = m_2/m_1$

$x_1(t) = A_1 \cos w_0 t$

$x_2(t) = A_2 \cos(w_0 t + \pi)$

以质心为界分为两段 k_1 k_2 l_1 l_2 [$k \propto \frac{1}{l}$]

$$\begin{cases} \frac{k_1}{k_2} = \frac{l_2}{l_1} = \frac{m_1}{m_2} \\ \frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{k} \end{cases} \Rightarrow \begin{cases} k_1 = \frac{m_1+m_2}{m_2} k \\ k_2 = \frac{m_1+m_2}{m_1} k \end{cases}$$

进而 $w_0 = \sqrt{\frac{k_1}{m_1}} = \sqrt{\frac{k}{\mu}}$ (μ 约化质量)

[凡是将两体问题转化为一体问题 则约化质量必会出现]

用能量语言描述 谐振子处于弹性势能谷，并作小振动

$k = \left(\frac{d^2E}{dx^2}\right)_0$ $w_0 = \sqrt{\frac{1}{\mu}\left(\frac{d^2E}{dx^2}\right)}$

[法2] 标准理论法

$$m_1 \frac{d^2x_1}{dt^2} = -k(x_1 - x_2)$$
$$m_2 \frac{d^2x_2}{dt^2} = -k(x_2 - x_1)$$

设稳定振动模式为

$x_1(t) = A_1 \cos wt$ $x_2(t) = A_2 \cos wt$

代入后整理得:

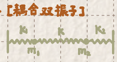
$$\begin{cases} -m_1 A_1 w^2 = -k(A_1 - A_2) \\ -m_2 A_2 w^2 = -k(A_2 - A_1) \end{cases} \Rightarrow \begin{cases} (k - m_1 w^2) A_1 - k A_2 = 0 \\ -k A_1 + (k - m_2 w^2) A_2 = 0 \end{cases}$$

为使振幅不为0 **[含非零解]** 则:

$$\begin{vmatrix} k - m_1 w^2 & -k \\ -k & k - m_2 w^2 \end{vmatrix} = 0 = (k - m_1 w^2)(k - m_2 w^2) - k^2$$

得 $w = \sqrt{\frac{k}{\mu}}$ 将 w 代回后: $\frac{A_1}{A_2} = -\frac{m_2}{m_1}$

▶ [耦合双振子]



$$m_1 \frac{d^2x_1}{dt^2} = -k_1 x_1 - k(x_1 - x_2)$$
$$m_2 \frac{d^2x_2}{dt^2} = -k_2 x_2 - k(x_2 - x_1)$$

$x_1 = A_1 \cos wt$ $x_2 = A_2 \cos wt$

$$\Rightarrow \begin{cases} (k_1 + k - m_1 w^2) A_1 - k A_2 = 0 \\ -k A_1 + (k_2 + k - m_2 w^2) A_2 = 0 \end{cases} \Rightarrow \begin{vmatrix} k_1 + k - m_1 w^2 & -k \\ -k & k_2 + k - m_2 w^2 \end{vmatrix} = 0$$

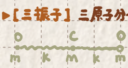
解出 $(k_1 + k - m_1 w^2)(k_2 + k - m_2 w^2) - k^2 = 0$ 即可

[特解: $k_1 = k_2 = k$ $m_1 = m_2 = m$]

$\Rightarrow (2k - m w^2)^2 - k^2 = 0$

有 $m w^2 = k$ 或 $3k$ $w_a = \sqrt{\frac{k}{m}}$ $w_b = \sqrt{\frac{3k}{m}}$ 可见存在两种振动模式

▶ [三振子] 三原子分子 简正振动模式



[解决方案] 设出 $x_1(t)$ $x_2(t)$ $x_3(t)$ ⟶ 列出动力学微分方程 ⟶ 设稳定解为同频同相位简谐型 → 振幅方程组 ⟶ 频率方程组 ⟶ 解出 w_a w_b w_c ⟶ 代回求对应振幅

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Keywords 关键词

Notes 笔记

弹簧有效质量

一维弹簧振子本征频率公式修正：$w_0 = \sqrt{\frac{k}{m+m^*}}$

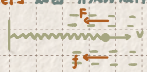
[离散质元法] N段 $m_1 = \cdots = m_N = \frac{m_0}{N}$

$k_1 = \cdots = k_N = Nk$

$m^* \approx 0.37 m_0$

§. 阻尼运动

▶ **[Def]** 如图 介质为液体或气体 物块受流体黏性力



$m\frac{d^2x}{dt^2} = F + f$

其中 $F = -kx$

f 一般表示为 $f = -\gamma\frac{dx}{dt}$

γ 为阻力系数 / 力阻 单位 $N \cdot s/m$

则有：$\frac{d^2x}{dt^2} + \frac{\gamma}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$

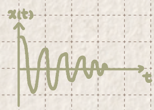
$\frac{\gamma}{m}\frac{dx}{dt}$ 称为阻尼项

▶ **[阻尼因数]** $2\beta = \frac{\gamma}{m}$

$\Rightarrow \frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + w_0^2 x = 0$

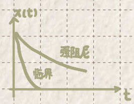
I. 弱阻尼衰减振动

$\beta^2 < w_0^2$ 时 $x(t) = Ae^{-\beta t}\cos(wt+\varphi_0)$ $w = \sqrt{w_0^2-\beta^2}$



II. 强阻尼衰减运动

$\beta^2 > w_0^2$ 时 $x(t) = C_1 e^{-(\beta-\beta_0)t} + C_2 e^{-(\beta+\beta_0)t}$ $\beta_0 = \sqrt{\beta^2 - w_0^2}$



III. 临界阻尼

$\beta^2 = w_0^2$ 时 $x(t) = (C_1 + C_2 t)e^{-\beta t}$

其中 (A, φ_0) (C_1, C_2) 由初始条件 (x_0, v_0) 确定

Summary 总结

Title 振动(下)

Date /

Keywords 关键词 | Notes 笔记 | Review 复习记录 / / / / /

§. 简谐量的保守性

▶ **[Theorem]** 简谐量微商仍为同频简谐

ex. $x(t) = A\cos(\omega t+\varphi_0)$

$v(t) = \omega A\cos(\omega t+\varphi_0+\frac{\pi}{2}) = B\cos(\omega t+\varphi_0')$

$a(t) = \omega B\cos(\omega t+\varphi_0'+\frac{\pi}{2}) = C\cos(\omega t+\varphi_0'')$

▶ **[Theorem]** 简谐量积分仍为同频简谐量

ex. $\int B\cos(\omega t+\varphi')\,dt = \frac{B}{\omega}\cos(\omega t+\varphi_0'-\frac{\pi}{2}) = A\cos(\omega t+\varphi_0)$

▶ **[Theorem]** 两个同频简谐量的合成仍为同频简谐量

ex. $x_1(t) = A_1\cos(\omega t+\varphi_{10})$ $\quad x_2(t) = A_2\cos(\omega t+\varphi_{20})$

$x(t) = x_1(t)+x_2(t) = A_1\cos(\omega t+\varphi_{10}) + A_2\cos(\omega t+\varphi_{20})$

$\Rightarrow x(t) = A\cos(\omega t+\varphi_0)$

$$\begin{cases} A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_{20}-\varphi_{10}) \\ \tan\varphi_0 = \dfrac{A_1\sin\varphi_{10}+A_2\sin\varphi_{20}}{A_1\cos\varphi_{10}+A_2\cos\varphi_{20}} \end{cases}$$

简谐量与复数

$x(t) = A\cos(\omega t+\varphi_0)$

$\updownarrow$

$\tilde{x}(t) = Ae^{i(\omega t+\varphi_0)}$

$\frac{d\tilde{x}}{dt} = i\omega Ae^{i(\omega t+\varphi_0)}$

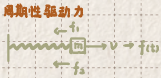
$= i\omega\tilde{x}$

故在考虑 $\tilde{x}$ 时 $\frac{d}{dt} \to i\omega$

$\int dt \to \frac{1}{i\omega}$

§. 受迫振动 & 共振

周期性驱动力



f_1 弹性力 $\quad f_2$ 阻尼力

$f(t) = F\cos\omega t$

$\frac{d^2x}{dt^2}m = f_1+f_2+f = (-kx)+(-\gamma\frac{dx}{dt})+F\cos\omega t \Rightarrow \frac{d^2x}{dt^2}+2\beta\frac{dx}{dt}+\omega_0^2x = C\cos\omega t$ $\quad \begin{cases} 2\beta = \frac{\gamma}{m} \\ \omega_0^2 = \frac{k}{m} \\ C = \frac{F}{m} \end{cases}$

[复数态求其定态解]

$\tilde{x}(t) = Ae^{i(\omega t+\varphi_0)}$

$\Rightarrow [(i\omega)^2 + 2\beta i\omega + \omega_0^2]\tilde{x} = Ce^{i\omega t}$

$\tilde{x}(t) = \frac{C}{(\omega_0^2-\omega^2)+i2\beta\omega}e^{i\omega t}$ $\quad \Rightarrow \begin{cases} A = \dfrac{C}{\sqrt{(\omega_0^2-\omega^2)^2+4\beta^2\omega^2}} \\ \varphi_0 = \arctan\dfrac{-2\beta\omega}{\omega_0^2-\omega^2} \end{cases}$ $\quad C = \frac{F}{m}$

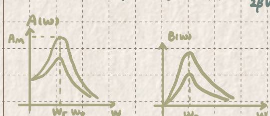
[Remark] $f_{受迫} = f_{外驱力}$

位移共振

条件: $\omega_r = \sqrt{\omega_0^2-2\beta^2}$ $\quad A_m = \frac{C}{2\beta\sqrt{\omega_0^2-\beta^2}}$

弱阻尼时: $\beta^2 \ll \omega_0^2$

$\omega_r \approx \omega_0$ $\quad A_m \approx \frac{C}{2\beta\omega_0}$



速度共振

$v(t) = B\cos(\omega t+\varphi_0')$

$B(\omega) = \omega A(\omega)$ $\quad \varphi_0'(\omega) = \varphi_0(\omega)+\frac{\pi}{2}$

能量分析

① 阻尼力 $f_2 = -\gamma v$ $\quad f_2v = -\gamma v^2 = -\gamma\omega^2A^2\cos^2(\omega t+\varphi_0')$

$P_2 = \frac{1}{T}\int_0^T f_2v\,dt = -\frac{1}{2}\gamma\omega^2A^2$

② 驱动力 $f(t) = F\cos\omega t$ $\quad fv = F\omega A\cos\omega t\cos(\omega t+\varphi_0')$

$P = \frac{1}{T}\int_0^T fv\,dt = \frac{1}{2}\gamma\omega^2A^2$

$P+P_2 = 0$

▶ **[Q值]** $Q = 2\pi\frac{\Delta E_0}{\Delta E}$ $\quad \Delta E_0$ 系统储能 $\quad \Delta E_0 = \frac{1}{2}kA^2$

ΔE 一周期内系统耗能值 $\quad \Delta E = \pi\gamma\omega A^2$

Q↑ 储能本领↑ 付出能量代价↓

$Q = \frac{k}{\gamma\omega}$

当 $\omega \approx \omega_0$ 时 $\omega_0 = \sqrt{\frac{k}{m}}$ $\quad Q = \frac{m\omega_0}{\gamma} = \frac{\sqrt{km}}{\gamma}$

Q↑ 峰值↑ 尖锐↑

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§. 非线性振动

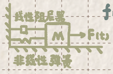
▶【单摆】 不作小角近似

$$\frac{d^2\theta}{dt^2}+\frac{g}{l}\sin\theta=0 \quad 即 \quad \frac{d^2\theta}{dt^2}+\frac{g}{l}\left(\theta-\frac{1}{6}\theta^3+\frac{1}{120}\theta^5-\cdots\right)=0$$

其近似解 $w \approx w_0\left(1-\frac{\theta_0^2}{16}\right)$ $\quad w_0=\sqrt{\frac{g}{l}}$ $\quad \theta_0$ 摆幅

考虑阻尼力和周期性外激励： $\frac{d^2\theta}{dt^2}+2\beta\frac{d\theta}{dt}+w_0^2\sin\theta = C\cos wt$

▶【混沌】



$f_1=-k_1x^2 \quad f_2=-0.05k_2v \quad F(t)=7.5k_3\cos t$

$$m\frac{d^2x}{dt^2}+0.05k_2\frac{dx}{dt}+k_1x^3=7.5k_3\cos t$$

⇒ 混沌

▶【非线性振动的物理特性】

(1) 本征频率与振幅有关

(2) 倍频与分数频响应

(3) 组合频率响应

(4) 频率俘获现象

(5) 跳跃和滞后的频应现象

§. 振动的合成

一维同频两个振动合成

$$\left.\begin{aligned}x_1&=A_1\cos(wt+\varphi_{10})\\x_2&=A_2\cos(wt+\varphi_{20})\end{aligned}\right\} \Rightarrow x(t)=A\cos(wt+\varphi_0)$$

$$\begin{cases}A^2=A_1^2+A_2^2+2A_1A_2\cos(\varphi_{20}-\varphi_{10})\\ \tan\varphi_0=\dfrac{A_1\sin\varphi_{10}+A_2\sin\varphi_{20}}{A_1\cos\varphi_{10}+A_2\cos\varphi_{20}}\end{cases}$$

令 $\Delta I = 2A_1A_2\cos\delta$ $\quad \delta\equiv\varphi_{20}-\varphi_{10}$ 相位差

if $\delta=2n\pi$ $\quad \Delta I=2A_1A_2 \quad A=A_1+A_2$ $\quad (A_m)$

if $\delta=(2n+1)\pi$ $\quad \Delta I=-2A_1A_2 \quad A=|A_1-A_2|$ $\quad (A_m)$

一维差频两个振动合成

▶[Def] 一质点沿一维x轴方向 同时含2种 不同频率简谐振动成分

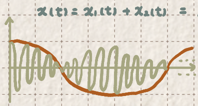
现简化设为 $x_1=A\cos w_1t$

$x_2=A\cos w_2t$

注意到 $\Delta w \ll w_1, w_2$ 低频变化的高频振动

$\cos(\frac{\Delta w}{2}t)$：低频包络(调幅)因子

$$x(t)=x_1(t)+x_2(t) = 2A\cos\left(\frac{\Delta w}{2}t\right)\cdot\cos\left(\frac{w_1+w_2}{2}t\right)$$



$w_b=2\times\frac{\Delta w}{2}=\Delta w$ $\quad f_b=\Delta f$ （T为调幅因子周期的一半）

此时 f_b 称为拍频，等于差频

Summary 总结

[Supplement]

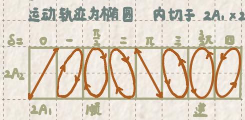
正交同频合成

$x(t)=A_1\cos(wt+\varphi_1)$

$y(t)=A_2\cos(wt+\varphi_2)$

$$\Rightarrow \frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}-2\frac{xy}{A_1A_2}\cos\delta=\sin^2\delta$$

运动轨迹为椭圆 内切于 $2A_1\times2A_2$ 矩形框



#正交非同频合成

李萨茹图形（略）

Chapter IX

Title 波动(上)

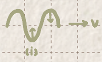
Date /

Keywords 关键词 | Notes 笔记 | Review 复习记录 / / / / /

§. 波与波函数

▶【横波】质量元振动方向与波传播方向正交

【纵波】质量元振动方向与波传播方向平行



▶【定态波】振源长时间持续稳定振动 $\longmapsto$ 波场中各点持续稳定振动

$$u(x,t) = A\cos(wt + \varphi_0(x))$$

【脉冲波】时间短，波包

▶【相速】波的传播速度：$v = \frac{\Delta x}{\Delta t}$

$\varphi_0(x) = \varphi_0 - w\frac{x}{v}$ （φ_0为原点振动初相位）

▶【平面简谐波函数】

$$u(x,t) = A\cos(wt + \varphi_0(x)) = A\cos[w(t - \frac{x}{v}) + \varphi_0]$$

• 时空双重周期性　时间 T　空间 $\lambda = \frac{v}{f}$

空间频率 $\tilde{v}$；空间角频率 k

$k = 2\pi\tilde{v} = \frac{2\pi}{\lambda}$　称 k, $\tilde{v}$ 为波数

公式表达：$u(x,t) = A\cos(wt - kx + \varphi_0) = A\cos[2\pi(\frac{t}{T} - \frac{x}{\lambda}) + \varphi_0]$

$v = f\lambda = \frac{w}{k}$

§. 波动方程

一维波动方程

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} = 0 \Rightarrow \begin{cases} u_1(x,t) = F(t - \frac{x}{v}) \\ u_2(x,t) = G(t + \frac{x}{v}) \end{cases}$$

$$\Rightarrow u(x,t) = C_1 F(t - \frac{x}{v}) + C_2 G(t + \frac{x}{v})$$

▶【传播因子】宗量 $t \mp \frac{x}{v}$ 包含时空变量

凡是以传播因子为宗量的函数均表示一种波动

三维波动方程

设波函数为 $\psi(\vec{r},t)$　位移函数 $u(\vec{r},t)$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} = 0$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \Rightarrow \nabla^2\psi - \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} = 0$$

$$\Rightarrow \psi(\vec{r},t) = \psi(t - \frac{\vec{v}\cdot\vec{r}}{v^2})$$

Summary 总结

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Title 波动(中)

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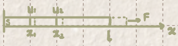
§. 弹性体的应变 应力

▶[杨氏模量] 

释加外力F 棒拉伸ΔL ⇒ ∀正截面 存在内应力F

$\frac{F}{S} \propto \frac{\Delta L}{L}$ ⇒ $\frac{F}{S} = E\frac{\Delta L}{L}$ 其中 $\frac{F}{S}$: 介质内单位面积上沿法向的弹性力 正应力/法向应力

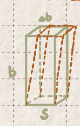
$\frac{\Delta L}{L}$: 相对伸长率 延伸率 E: 材料弹性参数 杨氏模量

设介质棒内位移函数 $u(x)$ 对应 $x \to x+\Delta x$ 段质元 位移量为 $u \to u+\Delta u$

$F(x) = ES\left(\frac{du}{dx}\right)$

if $\frac{du}{dx} = C$ 均匀应变 反之 非均匀应变

▶[切变模量]



F 切向形变 Δb ⇒ ∀正截面两侧存在切向弹性恢复力F

$\frac{F}{S} \propto \frac{\Delta b}{b}$ ⇒ $\frac{F}{S} = G\frac{\Delta b}{b}$ 其中: $\frac{F}{S}$ 介质内单位面积上沿切向的弹性力 切向应力/剪应力

$\frac{\Delta b}{b}$ 剪切应变 G 切变模量

泊松比 σ ⇒ $G = \frac{E}{2(1+\sigma)}$ 固体材料 σ: 0.3 ~ 0.4

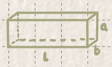
▶[体积模量] 流体几乎无切向弹性恢复力 $(G=0)$

对一静态流块/固体块 体积V

四周压强↑Δp 体积压缩 ΔV

$\Delta p = -K\frac{\Delta V}{V} = K\frac{\Delta\rho}{\rho}$ K: 体积模量

▶[弯曲与扭转] * 



横梁弯曲产生一个内力(应力)矩 以抗衡外力矩

当梁被弯曲 ⟼ 曲率半径 R

内力矩 $\Gamma = E\frac{I_s}{R}$ E: 杨氏模量 I_s 截面二次矩

I_s 取决于截面形状 矩形 $I_s = \frac{a^3b}{12}$ 实心圆 $I_s = \frac{\pi r^4}{4}$

*

一端固定圆柱体 自由端施加力偶 ⟼ 沿轴向扭转力矩

内力矩 $\Gamma = GI_p\frac{\alpha}{L}$ G: 切变模量 α: 离固定端距离为L的扭转角

I_p 有极转动惯量 取决于截面形状 对实心圆柱体 $I_s = \frac{\pi}{2}r^4$

§. 介质中的波速

导出波动方程

一根介质棒的一端敲击 激发纵波, 纵向位移 $u(x,t)$

$$F(x,t) = ES\frac{\partial u}{\partial x} \quad (1^*)$$

∀质元在 $x \mapsto x+dx$ 段 $dm = \rho S dx$

$$F(x+\Delta x) - F(x) = \Delta m\frac{\partial^2 u}{\partial t^2} \quad (2^*)$$

$$F(x+\Delta x) - F(x) = \Delta F = \frac{\partial F}{\partial x}\cdot\Delta x = ES\frac{\partial^2 u}{\partial x^2}\Delta x$$

$$E\frac{\partial^2 u}{\partial x^2} = \rho\frac{\partial^2 u}{\partial t^2} \quad \text{or} \quad \frac{\partial^2 u}{\partial x^2} - \frac{\rho}{E}\frac{\partial^2 u}{\partial t^2} = 0$$

* 波动是与位移函数对空间二级求导相联系的地位

▶[纵波相速公式] $v = \sqrt{\frac{E}{\rho}}$ E: 杨氏模量 ρ: 质量密度

▶[横波相速公式] $v = \sqrt{\frac{G}{\rho}}$

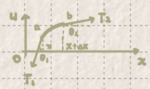
Keywords 关键词	Notes 笔记

▶【弦横波相速公式】 一根弹性弦 张力为T 线密度 η 局域横向扰动 $u(x,t)$

隔离一小段弧元 $\widehat{ab}$ 两端受力 T_1 T_2

$T\sin\theta_1 \approx T\left(\frac{\partial u}{\partial x}\right)_a$

$T\sin\theta_2 \approx T\left(\frac{\partial u}{\partial x}\right)_b \approx T\left(\frac{\partial u}{\partial x}\right)_a + T\left(\frac{\partial^2 u}{\partial x^2}\right)\Delta x$



则弦横向恢复力为 $\Delta F = T\sin\theta_2 - T\sin\theta_1 = T\left(\frac{\partial^2 u}{\partial x^2}\right)\Delta x$

$T\left(\frac{\partial^2 u}{\partial x^2}\right)\Delta x = \eta\,\Delta x\,\frac{\partial^2 u}{\partial t^2}$ 即 $\frac{\partial^2 u}{\partial x^2} - \frac{\eta}{T}\frac{\partial^2 u}{\partial t^2} = 0$

$\Rightarrow v = \sqrt{\frac{T}{\eta}}$

▶【声速公式】 气体中 $G=0$ 不存在横波

理想气体 $v = \sqrt{\frac{\gamma p_0}{\rho_0}} = \sqrt{\frac{\gamma RT}{\mu}}$ 即 $E_c = \gamma p_0$

其中 p_0 T ρ_0 为无波动时气体的压强、温度和密度 γ 气体绝热指数【定压比热容与定容比热容之比】

μ 为气体摩尔质量 $R = 8.314\ \mathrm{J/(mol\cdot K)}$

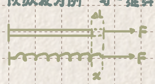
§. 能量与能流

▶【振动动能与弹性势能】

$\forall$ 介质元 $\Delta m = \rho\Delta V$ $u(x,t)$

$\Delta E_k = \frac{1}{2}\rho\left(\frac{\partial u}{\partial t}\right)^2\Delta V$

以纵波为例 与一维弹簧类比



$$\begin{cases} F = kx & \longleftrightarrow & F = ES\frac{1}{L}\Delta l \\ x & \longleftrightarrow & \Delta l \\ k & \longleftrightarrow & k_e = \frac{ES}{L} \end{cases}$$

$E_p = \frac{1}{2}kx^2 \longleftrightarrow E_p = \frac{1}{2}k_e(\Delta l)^2 = \frac{1}{2}E\left(\frac{\Delta l}{L}\right)^2 V$

$V \to \Delta V$ $\frac{\Delta l}{L} \to \frac{\partial u}{\partial x}$

$\Rightarrow \Delta E_p = \frac{1}{2}E\left(\frac{\partial u}{\partial x}\right)^2\Delta V$ 对横波 $\Delta E_p = \frac{1}{2}G\left(\frac{\partial u}{\partial x}\right)^2\Delta V$

▶【能量传输因子】 $u(x,t) = A\cos(wt - kx)$

$\left.\begin{aligned}\Delta E_k &= \frac{1}{2}\rho w^2A^2\Delta V\sin^2(wt-kx) \\ \Delta E_p &= \frac{1}{2}Ek^2A^2\Delta V\sin^2(wt-kx)\end{aligned}\right\}\Rightarrow$ 包含于 ΔV 中机械能 $\Delta E = \frac{1}{2}(\rho w^2 + Ek^2)A^2\Delta V\sin^2(wt-kx)$

$\left(v = \frac{w}{k} = \sqrt{\frac{E}{\rho}}\right)$ $= \rho w^2A^2\Delta V\sin^2(wt-kx)$

▶【平均能量密度】 $w(x,t) = \lim_{\Delta V\to 0}\frac{\Delta E}{\Delta V} = \rho^2w^2A^2\sin^2(wt-kx)$ 能量密度

$\bar{w} = \frac{1}{T}\int_0^T w(x,t)\,dt = \frac{1}{2}\rho w^2A^2$

▶【平均能流密度】 瞬时能流：单位时间内通过单位正截面的能量

能量传输速度 v $I = \bar{w}v = \frac{1}{2}\rho w^2A^2v$

Summary 总结

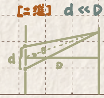
干涉

【一维】 设 $\left.\begin{aligned}u_1 &= \cos(kx - wt - \varphi) \\ u_2 &= \cos(kx - wt)\end{aligned}\right\}\Rightarrow u_1 + u_2 = 2A\cos\frac{\varphi}{2}\cos\left(kx - wt - \frac{\varphi}{2}\right)$

其 $I \propto \cos^2\frac{\varphi}{2} = \begin{cases} 1 & \varphi = m\pi \\ 0 & \varphi = (2m+1)\pi \end{cases}$ $\Delta x = m\lambda$ $\Delta x = (m+\frac{1}{2})\lambda$

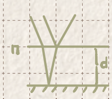
波程差：整数倍波长 ⇒ 相长干涉；半整数倍波长 ⇒ 相消干涉

【二维】 $d \ll D$ 波程差近似为 $d\sin\theta$



$d\sin\theta = m\lambda$ 明

$d\sin\theta = (m+\frac{1}{2})\lambda$ 暗



光程差 $2nd$

$2nd = (m+\frac{1}{2})\lambda$ 明

$2nd = m\lambda$ 暗

Chapter IX

Title 波动(下) Date /

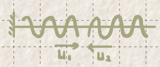
Keywords 关键词	Notes 笔记	Review 复习记录 / / / / /

§. 波的叠加和驻波

★ 在通常介质和通常波强度的条件下 有:

▶[波的独立传播定律] $\vec{u}(p,t) = \vec{u}_1(p,t) + \vec{u}_2(p,t)$

▶[驻波] 设两列波 f 相同 振动方向相同 v 相反



$u_1(x,t) = A_0\cos(wt-kx)$

$u_2(x,t) = A_0\cos(wt+kx)$

$$u(x,t) = u_1 + u_2 = 2A_0\cos kx\ \cos wt \Rightarrow \begin{cases} u(x,t) = A(x)\cos wt \\ A(x) = 2A_0\cos kx \end{cases}$$

驻波区别于行波的若干特点

(i) 波腹 波节 $\varphi_{10} = \varphi_{20} = 0$ 条件下 $\begin{cases} x_n = n\frac{\lambda}{2} & n = 0\ \pm1\ \pm2 \cdots \text{ 波腹 } A_m = 2A_0 \\ x_n = (n+\frac{1}{2})\frac{\lambda}{2} & \text{波节 } A_m = 0 \end{cases}$

(ii) 相邻波腹或节间 $\Delta x = \frac{\lambda}{2}$ 与初条件无关

(iii) 空间周期性 相邻两波节间区域视为一个单元 相邻单元振动相位差 π, 步调相反

(iv) 能量 $I = 0$ $\bar{w} = \bar{w}_1 + \bar{w}_2 = \rho w^2 A_0^2$

$$w_k = \frac{1}{2}\rho\left(\frac{\partial u}{\partial t}\right)^2 = 2\rho A_0^2 w^2\cos^2 kx\ \sin^2 wt$$

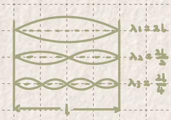
$$w_p = \frac{1}{2}E\left(\frac{\partial u}{\partial x}\right)^2 = 2EA_0^2 k^2\sin^2 kx\ \cos^2 wt$$

$\Rightarrow$ 波腹: $w_p(x_n,t) = 0$ $w_k(x_n,t) = 2\rho A_0^2 w^2\sin^2 wt$

波节: $w_k(x_n,t) = 0$ $w_p(x_n',t) = 2EA_0^2k^2\cos^2 wt$

▶[两端固定弦振动] 两端波节的驻波

L: $\lambda_n = \frac{2L}{n}$ $f_n = \frac{v}{\lambda_n} = n\frac{v}{2L} = nf_1$ f_1: 最低频率(基频) $\frac{v}{2L}$



▶[空气柱共振频率] 设空气柱长度为 L

① 两端开口: $\lambda_n = \frac{2L}{n}$ $f_n = n\frac{v}{2L}$

② 一端开口: $\lambda_n = \frac{4L}{(2n-1)}$ $f_n = (2n-1)\frac{v}{4L}$

§. 多普勒效应与激波

▶[纵向多普勒频移]

S 波源 S' 接收 发射频率 f u, v, u' ($S \to S'$ 正方向)

Δt 振动次数 $\Delta N = f\Delta t$

$L = V\Delta t$ $u = 0$

$L' = L - \Delta L = (v-u)\Delta t$ $u \neq 0$

波列以相对速度 $(v-u')$ 进入接收器 全部接收 $\Delta t' = L'/(v-u')$

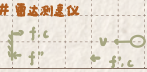
$$f' = \frac{\Delta N}{\Delta t'} = \frac{1-\frac{u'}{v}}{1-\frac{u}{v}}f \Rightarrow \begin{cases} \text{彼此靠近} & f' > f \\ \text{彼此远离} & f' < f \end{cases}$$

★ $v \gg u$ 时 $f' \approx (1-\frac{u'}{v})(1+\frac{u}{v})f$ $\Delta f = f'-f \approx \frac{u}{v}f$ ($u'=0$时)

★ u 改变波长 u' 改变接收时间

★ 不考虑相对论 对光波 $f' \approx (1+\frac{u_r}{c})f$ ($u_r = u-u' \ll c$)

雷达测速仪



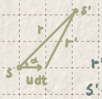
$$\left.\begin{aligned} f' &= (1+\frac{v}{c})f \\ f'' &= (1+\frac{v}{c})f' \end{aligned}\right\} \Rightarrow f'' = (1+\frac{v}{c})^2 f \approx (1+2\frac{v}{c})f \quad \Delta f = \frac{2v}{c}f = f_b \Rightarrow v = \frac{cf_b}{2f}$$

Keywords 关键词 | Notes 笔记

多普勒流速计

$$f'' = \left(\frac{v}{v+u}\right) f' = \left(\frac{v}{v+u}\right)\left(\frac{v-u}{v}\right) f$$
$$= \left(\frac{v-u}{v+u}\right) f$$

▶【多普勒频移普遍公式】

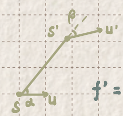


w　wdt

α　udt

$r'-r = -u\cos\alpha\, dt$

S'接收 $d\varphi = w dt$ 所需 $dt' \neq dt$



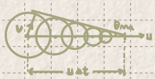
$$f' = \frac{v - u'\cos\beta}{v - u\cos\alpha} f$$ （连线速度）

$$dt' = dt + \frac{r'-r}{v} = \frac{v-u\cos\alpha}{v} dt$$

故 $w' = \frac{dt}{dt'} w = \frac{v}{v-u\cos\alpha} w \Rightarrow f' = \frac{v}{v-u\cos\alpha} f$

▶【激波与马赫锥】 物体运动速度大于波速

质点 Δt 掠过 $u\Delta t$ 起始时被激励球面波前传播半径 $v\Delta t < u\Delta t$



马赫锥

马赫数 $Ma = \frac{u}{v}$

马赫角 $\sin\theta_{Ma} = \frac{v}{u} = \frac{1}{Ma}$

§. 介质色散

▶【介质色散】不同波长或频率的波在同一介质中具有不同的传播速度

$v = \frac{w}{k}$　　$w(k)$：色散关系公式　　Ex：深水表面重力波的色散　$w = \sqrt{gk}$

▶【波包群速】 以调幅波为例：$u_1(x,t) = A\cos(w_1 t - k_1 x)$　　$u_2(x,t) = A\cos(w_2 t - k_2 x)$

调幅波函数 $u(x,t) = u_1 + u_2 = 2A\cos\left(\frac{\Delta w}{2}t - \frac{\Delta k}{2}x\right)\cos(\bar{w}t - \bar{k}x)$

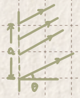
其中：$\bar{w} = \frac{1}{2}(w_1 + w_2)$　　$\bar{k} = \frac{1}{2}(k_1 + k_2)$

$\Delta w = w_1 - w_2 \ll \bar{w}$　　$\Delta k = k_1 - k_2 \ll \bar{k}$

波包推移速度 $v_g = \frac{dx}{dt} = \frac{\Delta w}{\Delta k}$ 即群速

Summary 总结

衍射



一束波通过宽为 a 的狭缝　缝宽为几个波长的数量级

$\Delta\varphi = k\Delta x = k\frac{n}{N} a\sin\theta \equiv \frac{n}{N}\varphi$

$$U_\theta = \sum_{n=0}^{N-1} u_n(\theta) = \sum_{n=0}^{N-1} A\cos\left(kx - wt + \frac{n}{N}\varphi\right)$$

近似地：$U_\theta = \sum_{n=0}^{N-1} \Delta n \cdot \frac{A}{N}\cos\left(kx - wt + \frac{n}{N}\varphi\right)$

$$I = I_0 \frac{\sin^2(\frac{\varphi}{2})}{(\frac{\varphi}{2})^2}$$

$\begin{cases} \varphi = 2m\pi \\ a\sin\theta = m\lambda \end{cases}$　强度极小　　$\theta = 0$　极大

Chapter x

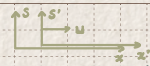
Title 相对论(上)

Date /

Keywords 关键词 | Notes 笔记 | Review 复习记录 / / / / /

§. 伽利略变换

▶【标准位形】惯性系 S S′ (y,z ∥ y′,z′ x x′重合)



当 $t=t'=0$ 时原点重合

则描述: S系中有一事件发生于 (x,y,z,t) ⟵ S中观察者记录

S′系中有一事件发生于 (x',y',z',t') ⟵ S′中观察者记录

$$\begin{cases} x' = x - ut \\ y' = y \\ z' = z \\ t' = t \end{cases} \quad (*)$$

伽利略变换满足 (*)

特征: 速度是相对的 $v_x' = v_x - u$ 加速度是绝对的 $a_x' = a_x$

Historical background

电磁学麦克斯韦理论不适用 ⟼ 以太? ⟼ 迈克耳孙-莫雷实验 不存在以太

§. 洛伦兹变换

▶【狭义相对论基本假设】

(i) 相对性原理: 物理定律在所有惯性系中都相同

(ii) 光速不变原理: 在所有惯性系中自由空间光速都有相同的值 c

▶【洛伦兹变换】 $S(x,y,z,t)$ $S'(x',y',z',t')$ 此处有事件对 (x_1,y_1,z_1,t_1) (x_2,y_2,z_2,t_2) Δxyz 空间差 Δt 时间差

$$\begin{cases} x' = \gamma(x-ut) \\ y' = y \\ z' = z \\ t' = \gamma(t - \frac{u}{c^2}x) \end{cases} \Rightarrow \begin{cases} \Delta x' = \gamma(\Delta x - u\Delta t) \\ \Delta y' = \Delta y \quad \Delta z' = \Delta z \\ \Delta t' = \gamma(\Delta t - \frac{u}{c^2}\Delta x) \end{cases}$$

其中 $\gamma = \frac{1}{\sqrt{1-(\frac{u}{c})^2}}$ $\beta = \frac{u}{c}$

特征: (i) $u<c$, 否则 γ 为虚数; (ii) $u \ll c$ 时 等价于伽利略变换

(iii) 存在不变值 $(\Delta S)^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$ 【事件对的空时间隔】

$$= (c\Delta t')^2 - (\Delta x')^2 - (\Delta y')^2 - (\Delta z')^2 = (\Delta S')^2$$

1° $(c\Delta t)^2 > (\Delta x)^2$ 类时 (timelike) $\Delta \tau = \sqrt{(\Delta t)^2 - (\Delta x/c)^2}$ 固有时间间隔

2° $(c\Delta t)^2 < (\Delta x)^2$ 类空 (spacelike) $\Delta \sigma = \sqrt{(\Delta x)^2 - (c\Delta t)^2}$ 固有空间间隔

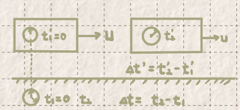
3° $(c\Delta t)^2 < (\Delta x)^2$ 类光 (lightlike)

Time Dilation 时间膨胀【动钟变慢】

S′ x' 时钟 $\Delta x' = 0$ $\Delta t'$

S中 $\Delta t = \gamma \Delta t' > \Delta t'$

⟹ S中观察者认为运动的时钟较慢



Lorentz Contraction 洛伦兹收缩【动尺缩短】

S′ $x_2' - x_1' = \Delta x' = l_0$

S $\Delta t = 0$ (同时测量) $\Delta x = \frac{\Delta x'}{\gamma}$

$l = l_0\sqrt{1-\frac{v^2}{c^2}}$

Summary 总结

Chapter

Title 相对论(下)　　Date /

Keywords 关键词 | Notes 笔记 | Review 复习记录 / / / / /

§. 相对论效应

▶【同时的相对性】

考虑 S 系中的同时事件 ($\Delta t = 0$)

$\Delta t' = -\gamma \frac{u^2}{c^2}\Delta x$

解释：S 地面和 S' 列车　分别在 A B 两点有两人　同时做动作

S 中光信号在 M (中点) 相遇

S' 中由于光速不变 仍在 M 相遇　显然 B' 光程长 B' 早发生

▶【因果律和信号速度】

$P(x_p\ t_p)$ 引起 $Q(x_Q, t_Q)$

因果关系：$V_S = \frac{x_Q - x_P}{t_Q - t_P} = \frac{\Delta x}{\Delta t}$　信息以 V_S 从 x_P 传到 x_Q

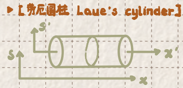
S'(u) 系中 $t_Q' - t_P'$ 为

$$\Delta t' = \gamma(\Delta t - \frac{u}{c^2}\Delta x) = \gamma\Delta t(1 - \frac{u}{c^2}\frac{\Delta x}{\Delta t}) = \gamma\Delta t(1 - \frac{uV_S}{c})$$

if $uV_S > c^2$　则时间次序将改变

当 $c > u > \frac{c^2}{V_S}$ 时　以 u 运动的观察者可看见不相关事件的颠倒

▶【劳厄圆柱 Laue's cylinder】



$\Delta t' = 0$

$\Delta t = \frac{u}{c^2}\Delta x > 0$　($\Delta x = \gamma\Delta x'$)

▶【光多普勒效应】

发射频率 ν_e　观察频率 ν_o　有 $\nu_o = \frac{1}{(1+\beta)\gamma}\nu_e$

退行红移 $\nu_o = \sqrt{\frac{1-\beta}{1+\beta}}\nu_e$　　$\nu_o = \sqrt{\frac{1+\beta}{1-\beta}}\nu_e$

§. 相对论运动学

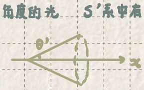
▶【基本方程】

邻近事件对洛伦兹变换

$$\begin{cases} dx' = \gamma(dx - u\,dt) \\ dy' = dy \\ dz' = dz \\ dt' = \gamma(dt - \frac{u}{c^2}dx) \end{cases} \Longrightarrow \begin{cases} V_x' = \dfrac{V_x - u}{1 - V_x\frac{u}{c^2}} \\ V_y' = \dfrac{V_y}{\gamma(1 - V_x\frac{u}{c^2})} \\ V_z' = \dfrac{V_z}{\gamma(1 - V_x\frac{u}{c^2})} \end{cases}$$

▶【光行差结论】

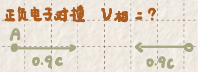
$\forall$ 角度的光　S' 系中有 图(1)



$\tan\theta' = \frac{V_y'}{V_x'}$　　在 S 系中 $\tan\theta = \frac{V_y}{V_x} = \frac{\sin\theta'\sqrt{1-\beta^2}}{\cos\theta' + \beta}$

§. 例题分析

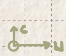
- 正负电子对撞　$V_{相} = ?$



以地面为 S　A 为 S'　$u = 0.9c$　$V_x = -0.9c$

$$V_x' = \frac{-0.9c - 0.9c}{1 + 0.9c\frac{0.9c}{c^2}} = -0.994c$$

- $V = ?$



$$\left.\begin{matrix} V_x' = 0 & V_x = u \\ V_y' = c & V_y = \frac{c}{\gamma} \end{matrix}\right\} \Rightarrow V = c$$

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Keywords 关键词	Notes 笔记

§9. 相对论动力学

▶【运动质量】 定义 $m \equiv m(v)$ $m_0 \equiv m(0)$

在S系中 $m(v_x) + m_0 = m_t(u)$

$$m(v_x) = \frac{m_0}{\sqrt{1-\beta^2}} \equiv \gamma m_0 \quad (\beta \equiv \frac{v_x}{c})$$

m_0为静止质量 对 $m_t(u)$ 在S'系 $m_{t0} = 2m(u) = 2\gamma(u) m_0$

$\Longrightarrow m_t(u) = \gamma m_{t0}$

▶【动量】 $\vec{p} = m\vec{v} = \dfrac{m_0 \vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$

▶【动能】 $E_k = m_0 c^2 \left[\dfrac{1}{\sqrt{1-v^2/c^2}} - 1\right]$

▶【质能】 引入质能 $E = mc^2$ 表达 动能与静止能量之和

$$E^2 = c^2p^2 + m_0^2c^4 \quad // \quad E = \sqrt{c^2p^2 + m_0^2c^4}$$

对 $m_0 = 0$ 粒子 $E = cp$ $p = \frac{E}{c}$

▶【加速度】 $\vec{a} = \dfrac{\vec{F}}{m} - \dfrac{\vec{F}\cdot\vec{v}}{mc^2}\vec{v}$

Summary 总结